EFFECT OF INTERNAL HEAT GENERATION ON RAYLEIGH- BENARD CONVECTION IN FERROMAGNETIC LIQUIDS

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Abstract: Buoyancy driven convection in Newtonian ferromagnetic liquids is studied with a heat source. The basic temperature gradient here is non-uniform. The critical eigen values are obtained using Galerkin technique. The influence of various magnetic and nonmagnetic parameters on the onset of ferroconvection is analyzed. It is found that effect of heat source destabilizes the system and the heat sink is to stabilize the system. Buoyancy magnetization parameter destabilizes the system both in presence/absence of heat source/sink.

Introduction

Thermal convection in ferromagnetic liquids has been studied by many authors (Finlayson [1], Gotoh and Yamada [2], Kaloni and Lou [3], Sekhar and Rudraiah [4], Sunil and Mahajan [5], Sekhar G. N. and Jayalatha G. [6]). Most of the papers do not consider non-uniform heat source. This is considered in this paper.

Mathematical Formulation

Consider an infinite horizontal layer of ferromagnetic liquid confined between two plates at a distance d apart. A Cartesian co-ordinate system is taken with the lower plate in the xy-plane and z-axis vertically upwards. The lower plate at $z = 0$ and the upper plate at $z = d$ are maintained at constant temperatures $T_0 + \Delta T$ and T_0 respectively (see Figure 1). A uniform magnetic field H_0 is applied in the vertical direction.

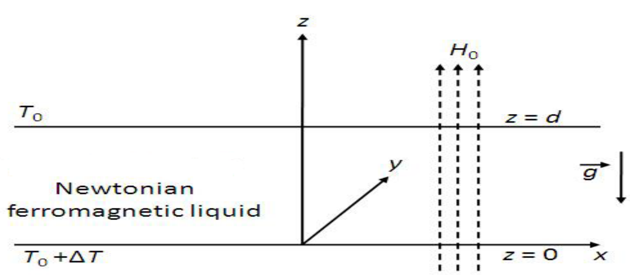

Figure 1: **Schematic of the problem.**

The governing equations for ferromagnetic liquid and heat source are:

$$\mathrm{R}M_1 a^2 \, \mathrm{D} \, \mathrm{f}(z) \, [T - D\Phi] + g_1(z) \left(D^2 - a^2\right)^2 w + 2\mathrm{D} \, g_1(z)\left(D^2 - a^2\right) \mathrm{D}w + D^2 g_1(z) \left(D^2 + a^2\right)\mathrm{w} - \mathrm{R} \, a^2 \, \mathrm{T} = 0 \tag{1}$$

$$\left(D^2-a^2\right)T+R_I T-g_2(z)w=0 \tag{2}$$

$$\left(D^2-M_3a^2\right)\Phi-DT=0 \tag{3}$$

Where

$$f(z) = \quad f(z)=\cos\left(\sqrt{R_I}\,z\right)-\cot\left(\sqrt{R_I}\right)\sin\left(\sqrt{R_I}\,z\right)$$
$$g_1(z)=\left[1+f(z)\left\{\delta_r-\frac{\delta_H k_I}{1+\chi_m}\right\}\right]^{-I}$$
$$g_2(z)=-\left[\sqrt{R_I}\ \mathrm{Sin}\sqrt{R_I}\,z+\sqrt{R_I}\cot\left(\sqrt{R_I}\right)\cos\left(\sqrt{R_I}\,z\right)\right]$$

The dimensionless parameters appearing in the above equations are the following:

$\Pr=\dfrac{\mu_0}{\rho_0\chi}$ (Prandtl number),

$R=\dfrac{\alpha\rho_0 g d^3\Delta T}{\mu_0\chi}$ (Rayleigh number),

$M_1=\dfrac{\mu_0 k_1{}^2\Delta T}{\alpha g\rho_0(1+\chi_m)d}$ (Buoyancy Magnetization number),

$R_I=\dfrac{Q_1 d^2}{k\Delta T}$ (Internal Rayleigh number),

$RM_1=\dfrac{k_1{}^2\Delta T d^2}{(1+\chi_m)\chi}$ (Magnetic Rayleigh number),

$M_3=\dfrac{1+M_0/H_0}{(1+\chi_m)}$ (Non- Buoyancy Magnetization number)

We consider RIFA boundary combinations in studying the problem. Using the Galerkin technique, the following expression for the Thermal Rayleigh number R is obtained in the form :

$$R=\frac{(X_1+X_2+X_4)\left[X_9-M_3a^2X_{10}\right](X_{71}+X_{72}R_I)}{\left[(M_1a^2X_4-a^2x_6)(M_3a^2X_8X_{10}-X_8X_9)+M_1a^2X_5X_8X_{11}\right]}, \tag{4}$$

where

$$X_1=\left\langle g_1w_1\left(D^2-a^2\right)^2w_1\right\rangle, X_2=2\left\langle Dg_1w_1\left(D^2-a^2\right)Dw_1\right\rangle, X_3=\left\langle D^2g_1w_1\left(D^2+a^2\right)w_1\right\rangle$$
$$X_4=\langle w_1Df(z)T_1\rangle, X_5=\langle w_1Df(z)D\varphi_1\rangle, X_{6=}\langle w_1T_1\rangle, X_{71}=\left\langle T_1D^2T_1-a^2T_1^2\right\rangle,$$
$$X_{72}=\left\langle T_1^2\right\rangle, X_8=\langle g_2(z)T_1w_1\rangle,\ X_9=\left\langle\varphi_1D^2\varphi_1\right\rangle, X_{10}=\left\langle\varphi_1^2\right\rangle \text{ and } X_{11}=\langle\varphi_1DT_1\rangle$$

Results and Discussion

Present study deals with a linear stability analysis of thermal convection in temperature and magnetic field sensitive Newtonian ferromagnetic liquid. The principle of exchange of stabilities is shown to be valid and the critical eigenvalues for stationary convection are obtained using Galerkin technique for RIFA boundary combinations. The influence of various magnetic and nonmagnetic parameters on the onset of convection has been analyzed. The critical eigenvalues for stationary convection are obtained using Galerkin technique. It is found that effect of heat source destabilizes the system and the effect of heat sink is to stabilize the system. Buoyancy magnetization parameter destabilizes the system both in presence/absence of heat source/sink.

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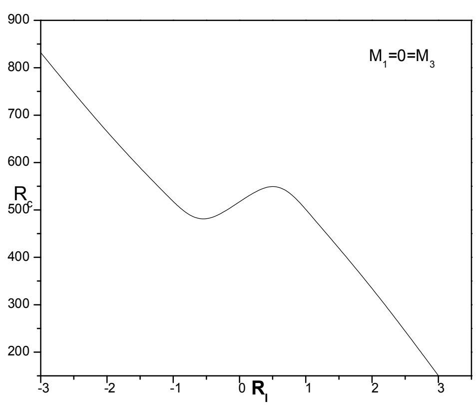

Figure2:Critical Rayleigh Number R_C verses internal Rayleigh number R_I

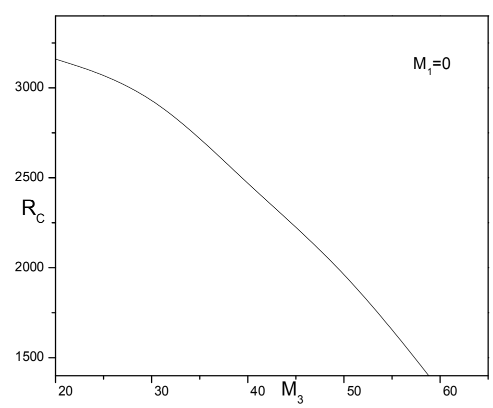

Figure3:Plot of R_c verses non buoyancy magnetization number M_3