ANALYTICAL STUDY OF HEAT TRANSPORT BY RAYLEIGH - BÉNARD CONVECTION IN A RADIATING LIQUID

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Abstract: Analytical study of Nonlinear Rayleigh - Bénard convection with thermal radiation is presented. Optically thin (transparent medium) and optically thick fluid media (opaque medium) are considered.Coupled Ginzburg – Landau equations are derived from the generalized five mode Lorenz model arising in the problem. An analytical expression for the Nusselt number is obtained using the coupled Ginzburg–Landau equations. The effect of radiation related parameters is to delay onset and hence to diminished convection.

Keywords: Nonlinear Rayleigh - Bénard convection, Thermal radiation, Generalized five-mode Lorenz model, Coupled Ginzburg–Landau model, Nusselt number.

Introduction

The convective instability problems for radiating liquids received great attention inthe past due to their implications in astrophysical and geophysical applications and in other applications such as solar collectors ([1]).The Rayleigh- Bénard instability problem involve only two modes of heat transfer,viz., conduction and convection. Radiative heat transfer is important in physicalsystems because of its stabilizing effect ([2] –[4]). Goody ([5]) estimated the radiative transfer effects in the natural convection problem with free boundaries using a variational method. Goody's radiative transfer model has been extended and modified by many researchers by taking into account the effects of magnetic field, rotation and fluid non-grayness ([6] –[10] and references therein). Motivated by the meteorological applications, Larson ([11]) studied the linear and nonlinear stability of an idealized radiative-convective model due to Goody ([5]).

There are number of individual works available on either Ginzburg-Landaumodel ([12], [13] and references therein)or Lorenz model ([14] – [16] and references therein) that address nonlinear convection. The two models were connected by Siddheshwar and Titus ([17])in non-radiating liquids.

The effect of thermal radiation on heat transport by Rayleigh-Bénard convection is investigated in this paper using generalized Lorenz model from which the Ginzburg-Landau model is arrived to facilitate the getting of an anaytical expression for the Nusselt number.

Mathematical formulation

By assuming a penta-modal representation for the considered conduction-convection-radiation problem in the form:

$$\left.\begin{aligned}\psi(x,z,\tau) &= (A_1(\tau)\sin\pi\alpha X - A_2(\tau)\cos\pi\alpha X)\sin\pi z,\\ \Theta(x,z,\tau) &= (B_1(\tau)\cos\pi\alpha X + B_2(\tau)\sin\pi\alpha X)\sin\pi z - C(\tau)\sin 2\pi z,\end{aligned}\right\}, \tag{1}$$

the governing nonlinear partial differential equations with variable coefficientsresults in the five-mode Lorenz model as follows:

$$\frac{dA_1}{d\tau_1} = \Pr(B_1 - A_1). \tag{2}$$

$$\frac{dA_2}{d\tau_1} = \Pr(B_2 - A_2), \tag{3}$$

$$\frac{dB_1}{d\tau_1} = r_E c_A A_1 - c_B B_1 - A_1 C, \tag{4}$$

$$\frac{dB_2}{d\tau_1} = r_E c_A A_2 - c_B B_2 - A_2 C, \tag{5}$$

$$\frac{dC}{d\tau_1} = A_1 B_1 + A_2 B_2 - bC, \tag{6}$$

where

$$L_1 = \chi_1 \left[\left(\frac{2\chi_1}{\tau_1} + \frac{1}{2}\sqrt{3+3\chi_1} \right) \sinh(\frac{\tau_2}{2}) + \cosh(\frac{\tau_2}{2}) \right]^{-1},$$

$$L_2 = \frac{L_1}{\chi_1} \left[\left(\frac{1}{2}\sqrt{3+3\chi_1} \right) \sinh(\frac{\tau_2}{2}) + \cosh(\frac{\tau_2}{2}) \right],$$

$$\tau_1 = \eta_1^2 \tau, \quad \tau_2^2 = 3k_a^2 d^2 (1+\chi_1), \quad \chi_1 = \frac{4\pi Q_1}{3\kappa k_a S_r}, \quad Q_1 = \frac{4S_c}{\pi} T_a^2,$$

$$\eta_1^2 = \pi^2(\alpha^2+1), \ r_E = \frac{R_E}{R_{E_c}}, \quad R_{E_c} = \frac{\eta_1^6}{\alpha^2}, \quad c_A = \left(\frac{8\pi^2}{\tau_2(\tau_2^2 + 4\pi^2)} \right) L_1 \sinh(\frac{\tau_2}{2}) + L_2,$$

$$c_B = 1 + \chi_1 \left(\delta_{k1} \frac{\tau_2^2}{\eta_1^2(1+\chi_1)} + \delta_{k2} \right), \ b = \left[4\pi^2 + \chi_1 \left(\delta_{k1} \frac{\tau_2^2}{(1+\chi_1)} + 4\pi^2 \delta_{k2} \right) \right] \eta_1^{-2},$$

$$\delta_{k1} = \begin{cases} 1, & k=1 \\ 0, & k \neq 1 \end{cases} \text{(Transparent case)}, \ \delta_{k2} = \begin{cases} 1, & k=2 \\ 0, & k \neq 2 \end{cases} \text{(Opaque case)}.$$

Eliminating B_1, B_2 and C from the Lorenz model, we get the coupled Ginzburg-Landau model in the form:

$$\frac{dA_1}{d\tau_1} = \left[b(\frac{c_B}{\Pr}+1) \right]^{-1} \left\{ b(r_E c_A - c_B) A_1 - A_1^3 - A_1 A_2^2 \right\}, \tag{7}$$

$$\frac{dA_2}{d\tau_1} = \left[b(\frac{c_B}{\Pr}+1) \right]^{-1} \left\{ b(r_E c_A - c_B) A_2 - A_2^3 - A_2 A_1^2 \right\}. \tag{8}$$

The Nusselt number as a function of Rayleigh number is obtained in the following form:

$$Nu(\tau_1) = 1 + \frac{2}{r_E} \left[\frac{1}{L_1 \cosh(\tau_2/2) + L_2} \right] F\left(A1(\tau_1), A2(\tau_1)\right), \tag{9}$$

where $F\left(A1(\tau_1), A2(\tau_1)\right)$ is a lengthy analytical expression involving the parameters of the problem.

Results and Discussion

Five Fourier modes are used in the nonlinear analysis of convection and an elegant looking generalized Lorenz model for convection in a Newtonian liquid with thermal radiation is arrived at by meticulous rearrangement of the governing equations and by scaling of the amplitudes. The coupled Ginzburg-Landau equations are derived from the five- mode Lorenz model which is then used to derive the analytical expression for the Nusselt number.

Heat transport is quantified in the form of a Nusselt number, *Nu,* to find the effect of Prandtl number, *Pr,* conduction-radiative parameter , χ_1, absorptivity parameter, τ_1, and Rayleigh number, R_E, on it. The Nusselt number is shown to decrease with increases in χ_1 and τ_1, in both transparent and opaque cases.The expenditure of part of the energy of the system in inducing radiation from the liquid medium is responsible for the delay in onset of convection in radiating liquids leading to a diminished-convection situation. All other effects on Nusselt number are classical.

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