Thermal Transport of Electromagnetohydrodynamic flow of Casson-nano fluid through a Porous Microtube under the effect of Streaming Potential

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ABSTRACT

Thermal transport characteristics of casson-nano fluid through a porous microtube is analyzed under the effect of streaming potential and constant pressure gradient with electrokinetic effect associated with applied magnetic field. An analytical solution of the velocity and temperature distribution of Casson-nano fluid through the porous microtube related to combining effects of electromagnetohydrodynamics forces under the effect of streaming potential have been obtained.The significant influences of various non-dimensional parameters on velocity and temperature profiles are discussed in this study. Also, it is revealed the impact of nano particles on flow transport and heat transfer phenomenon. Furthermore, the Nusselt number is calculated analytically. The variations of pertinent parameters such as Hartmann number, Darcy number,casson parameter, volume friction parameter of nanoparticles, joule heating parameter are delineated graphically and discussed in details

Keywords: Electromagnetohydrodynamic flow, Microtube, Porous Medium, Casson fluid, nano fluid.

1. INTRODUCTION

To improve more advance microfluidic devices, microfluidic technology have being undertaken in so many fields such as bio chemical analysis instruments, micro-heat exchanger, biomedical systems, lab-on-chip devices, micropumps, micro-turbines etc [1,2]. There are number of researchers who have investigated the hydrodynamic and thermo-fluidic transport of the electroosmotic flow through a microchannel. Further, the measurements of streaming potential is also investigated by so many researchers. Heat transfer of a nano-fluid flow through a microchannel with trigular/rectangular ribs is studied by Behnampour et al [3]. Based on the new KKL (Koo-Kleinstreuer-Li) model Li and Kleinstreuer [4] investigated the thermal performance of nanofluid flow through a trapezoidal microchannel which is induced by the Brownian micro-mixing motion. Yang et al [5] represented the electroosmotic flow and streaming potential through a heterogeneous microtube with nonuniform zeta potentials The discussed about the streaming potential and the flow phenomenon due to electroosmosis. An analytical solution of streaming potential and the periodical electroviscous effects in a pressure driven flow through a uniform microchannel is done by Gong et al [6]. An experimental investigation is provided by Lu et al [7] on the flow field effect on the EDL of the microchannel with the streaming potential.

The previous studies are focused only on the electroosmotic flow. Recently there are so many analytical and experimental investigations are going on the electromagnetohydrodynamic (EMHD) flows to developed EMHD micropumps. The flow control phenomenon in a microchannel with the combined influence of electromagnetohydrodynamic effects is studied by Chakraborty and Paul [8]. Further thermal characteristics of a EMHD flow through microchannel under constant wall heat flux conditions is analysed by Chakraborty et al.[9] . An comprehensive analysis of heat transfer of nano-fluid through a microtube under the streaming potential effects is done by Zhao et al [10].

From the motivation of the above study, we want to analyse the nanofluid flow through a porous microchannel under streaming potential effects. The main objective of this investigation is to study the velocity and heat transport for Casson-nano fluid flow in porous microtube under the effect of streaming potential with electrokinetic force associated with constant pressure gradient and applied magnetic field. The analytical solutions of velocity and temperature distribution for the Casson-nano fluid flow are derived using the various dimensionless parameters. The transport phenomenon of this fluid flow has been examined under the combined effect of the electric double layer and applied magnetic field as well as streaming potential effect. The velocity and temperature expression are obtained. Furthermore, the streaming potential and nusselt number are obtained analytically and investigated the variations of nusselt number and streaming potential with various non-dimensional parameters graphically.

2. MATHEMATICAL MODEL AND PROBLEM DESCRIPTION

The EMHD flow of a Casson-nano fluid in a porous microtube under the effect of streaming potential is considered along axial direction of the microtube. The physical sketch of this flow problem is depicted in Fig 1. This circular microtube is replete by a porous medium where R and L denote the radius of the microtube and length of the microtube respectively. The electrolyte solution generates a EDL near to the boundary wall and the induced electric field in opposite axial direction creates a electroosmotic flow. In the radial direction, the electromagnetic field is induced by the applied magnetic field and electric field. The polar, cylindrical coordinate (r,θ,z) is considered to describe the governing equations of this flow problem.

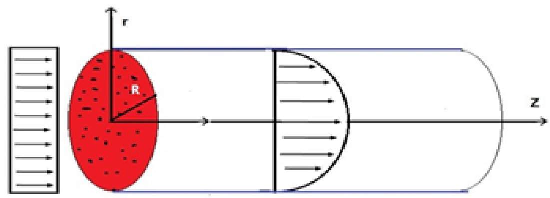


Figure 1: Schematic diagram of circular microtube with porous medium

2.1.Governing Equation and Velocity Distribution Analysis

In this flow problem casson- nano fluid is considered where the base fluid is the non-Newtonian casson fluid. The casson fluid sample plays Newtonian and non-Newtonian both fluid behavior. The constitutive equation for casson fluid model can be followed by

$$\tau_{ij} = \begin{cases} \left(\mu_c + \frac{P_s}{\sqrt{2\pi}}\right) 2e_{ij}\,, & \pi > \pi_0 \\ \left(\mu_c + \frac{P_s}{\sqrt{2\pi_0}}\right) 2e_{ij}\,, & \pi < \pi_0 \end{cases} \tag{1}$$

Where the yield stress of the fluid is $P_s = e_{ij}e_{ij}$ and e_{ij} indicates the deformation rate at (i,j) th component. π is the product of the component of deformation rate with itself, π_0 is a critical value of this product based on the non-Newtonian fluid model, μ_c represents the plastic dynamic viscosity of the non-Newtonian fluid. Then from (1) it is obtained that $(for\ \pi < \pi_0)$

$$\tau_{ij} = \mu_c\left(1 + \frac{1}{\beta}\right) 2e_{ij} \tag{2}$$

where $\beta = \mu_c\sqrt{2\pi_0}/P_s$ is the casson parameter.

Then the effective viscocity of the nano fluid is given by

$$\mu_{eff} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{3}$$

Where the μ_f is the viscosity of the base fluid and ϕ is called the volume friction of the nanoparticles.The flow problem is considered as thermally fully developed, steady, symmetrical in azimuthal direction flow under cylindrical co-ordinate system. Therefore the velocity component is only in z-direction, which depends on r.The simplified form of the momentum conservation equation along the *z*-direction, assuming a hydrodynamically fully developed flow of a casson-nano fluid is written as:

$$-\frac{\partial p}{\partial z}+\mu_{eff}\left(1+\frac{1}{\beta}\right)\left(\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}\right)-\mu_{eff}\frac{u}{K}+\rho_e E_s+\sigma_e B_0 E_1-\sigma_e B_y^2 u=0 \quad (4)$$

Where β is Casson parameter, K denoted the permeability of the porous medium andE_0 is the axial component and E_1 is the lateral transverse component of applied electrical force. Due to small ratio of the diameter and length of the microtube, although the body force may exist in the radial or angular direction and the velocity in these directions is much smaller than the axial direction. Therefore only one momentum equation is considered in the axial direction. The boundary conditions are derived as follows:

$$u(r=R)=0 \; and \; \frac{du}{dr}(r=0)=0 \quad (5)$$

The net charge density ρ_e in the EDL depends on the EDL potential ψ and it is given by [11]:

$$\rho_e=-\varepsilon\kappa^2\psi \quad (6)$$

The EDL potential ψ is well constructed by the Poisson-Boltzmann equation with the assumptions (i) the permittivity of the fluid is not influenced by the field strength and it is constant; (ii) the ions are considered as point charges. Then under the Debye- H$\ddot{u}$ckle approximation the EDL potential distribution is obtained as :

$$\psi=\psi_w I_0(\kappa r)/I_0(\kappa R) \quad (7)$$

Where H is the half height of the channel, ζ be the constant zeta potential and $1/\kappa$ be the thickness of the EDL where $\kappa=ez\left(\frac{2n_0}{\varepsilon\, k_B\, T_a}\right)^{\frac{1}{2}}$ is called Dybye- H$\ddot{u}$ckle parameter where n_0 is the ion density (in molar unit) of the liquid, z is the valance of ions of the solution, k_B is the Boltzmann constant, and T_a is the absolute temperature, ε is permittivity of the fluid and $\omega=\kappa R$ is called the normalized reciprocal thickness of the EDL which representing the ratio of half height of microchannel to Debye length (i.e. $1/\kappa$).

Let us introduce the following dimensionless variables and parameters:

$$u^* = \frac{u}{u_{HS}}\ , r^* = \frac{r}{R}\ , G = \frac{\left(-\frac{\partial p}{\partial z}\right)R^2}{\mu_f u_{HS}}\ , \psi^* = \psi\left(\frac{ez}{k_B T}\right), \psi_w^* = \psi_w\left(\frac{ez}{k_B T}\right),$$

$$E_S^* = \frac{E_S}{E_0}\ , Da = \frac{K}{R^2}\ , Ha = B_0 R\sqrt{\frac{\sigma_e}{\mu_f}}\ , u_{HS} = \frac{-\varepsilon E_0 k_B T}{ez\mu_f}\ , S = \frac{E_1 R}{u_{HS}}\sqrt{\frac{\sigma_e}{\mu_f}}$$

Where u_{HS} be the reference velocity,G be the non-dimentional pressure gradient. $Ha$is the Hartmann number which is indicating the strength of the applied magnetic field B_0 and Da is the Darcy number. S represents the strength of the transverse electric E_1field. The non-dimensional form of the equation (13) is reduced by:

$$\frac{\left(1+\frac{1}{\beta}\right)}{(1-\phi)^{2.5}}\left(\frac{d^2u^*}{dr^{*2}} + \frac{1}{r^*}\frac{du^*}{dr^*}\right) + \omega E_S^*\psi^* - \left(Ha^2 + \frac{1}{Da(1-\phi)^{2.5}}\right)u^* + HaS - G\ = 0 \qquad (8)$$

Boundary condition (5) is reduced as follows:

$$u^*(r^* = 1) = 0\ and\ \frac{du^*}{dr^*}(r^* = 0) = 0 \qquad (9)$$

Equation (8) is analytically solved analytically based the boundary conditions (9) to obtain the exact solutions which is written as

$$u^*(r^*) = \frac{I_0(Mr^*)}{I_0(M)}\left\{\frac{A\ E_S^* M^2}{M^2-\omega^2} I_0(\omega) + B\right\} - B - \frac{A\ E_S^* M^2}{M^2-\omega^2} I_0(\omega\ r^*) \qquad (10)$$

where $A = \frac{(1-\phi)^{2.5}\omega^2\psi_w^*}{(1+1/\beta)\ I_0(\omega)}$, $B = \frac{(G-HaS)(1-\phi)^{2.5}}{(1+1/\beta)}$, $M = \frac{\{(1/Da)+Ha^2(1-\phi)^{2.5}\}}{(1+1/\beta)}$

The above expression of velocity function depends on different physical parameters like Hartmann number Ha, Casson parameter β, Darcy number Da, volume fraction parameter ϕ etc.

2.2 Streaming Potential:

The streaming potential is obtained from the following expression:

$$i_{net} = I_{St} + I_{Co} = 0 \qquad (11)$$

Which tells that the net ionic current through the tube section equal to zero. Where I_{St} is Streaming current and I_{Co} is the Conduction current. It is assumed that the ions are moved within the EDL in the electrolyte solution. Then the streaming current and Conduction current are written as:

$$I_{St} = 2\pi \int_0^R z\, e\, (n_+ - n_-) ur\, dr \,;\, I_{Co} = 2\pi \int_0^R z\, e\, (n_+ + n_-) \left(\frac{ezE_s}{f}\right) r\, dr \qquad (12)$$

Here $f = 2n_0 e^2 z^2/\sigma_b$ is called the ionic friction factor where σ_b is the bulk ionic conductivity and $n_\pm = n_0 \exp(\mp \frac{ez\psi}{k_B T_{av}})$ is the ionic number concentration of positively and negatively charge species. By substituting all the dimensionless variables in (18) the following dimensionless form is obtained:

$$\int_0^1 \psi^* u^* r^* dr^* - \alpha^* E_s^* \int_0^1 r^* dr^* = 0 \qquad (13)$$

Where $\alpha^* = \mu\sigma_b/2n_0\varepsilon k_B T_{av}$ represents the non-dimensional conductivity parameter. And finally the non-dimensional streaming potential is given by $E_s^* = \frac{2\tau_1^*}{\alpha^* - 2\tau_2^*}$ (14)

Here τ_1^* and τ_2^* is obtained from the following expressions:

$$\tau_1^* = \int_0^1 \frac{\psi_w^* A\, I_0(\omega r^*)}{I_0(\omega)} \{\frac{I_0(\omega) I_0(Mr^*)}{(M^2-\omega^2) I_0(M)} - \frac{M^2 I_0(\omega r^*)}{(M^2-\omega^2)}\} r^* dr^*$$

$$\tau_2^* = \int_0^1 \frac{\psi_w^* B\, I_0(\omega r^*)}{I_0(\omega)} \{\frac{I_0(Mr^*)}{I_0(M)} - 1\} r^* dr^* \qquad (15)$$

2.3 Temperature distribution and heat transfer analysis

The governing equations of energy equation for thermally fully developed flow is expressed as

$$(\rho C_P)_{eff} u \frac{\partial T}{\partial z} = k_{eff} \{\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r}) + \frac{\partial^2 T}{\partial z^2}\} + S_j \qquad (16)$$

where $(\rho C_P)_{eff}$ is the effective heat capacitance of the nanofluid and which is calculated by the following expression: $(\rho C_P)_{eff} = \phi(\rho C_P)_P + (1-\phi)(\rho C_P)_f$ (17)

Where $(\rho C_P)_P$ and $(\rho C_P)_f$ are called the heat capacities per unit volume of the solid nano particles and base fluid respectively. S_j is the volumetric heat generation due to joule heating respectively. Where $S_j = \sigma_e E_s^2$ and k_{eff} is the thermal conductivity of the fluid, T is the local temperature of the liquid. Introducing the non-dimensional temperature $T^* = \frac{k_f(T-T_w)}{q_w R}$ where T_w is the channel wall temperature and q_w is the constant wall heat flux. Further, for thermally fully developed flow under imposed constant wall heat flux, one may write

$\frac{\partial T}{\partial z} = \frac{dT_w}{dz} = \frac{dT_b}{dz} = constant$ and $\frac{d^2T}{dz^2} = 0$ in which T_b is the bulk mean temperature. The overall energy balance of an elementary control volume of the fluid with the length of duct dx gives the following expression:

$$(\rho C_P)_{eff} \pi R^2 u_m \frac{dT_b}{dz} = 2\pi R q_w + \sigma_e E_s^2 \pi R^2 \qquad (18)$$

Then the bulk mean temperature gradient is written as $\frac{dT_b}{dz} = \frac{\gamma_1}{(\rho C_P)_{eff}} = constant.$ (19)

where $\gamma_1 = \frac{2q_w + \sigma_e E_s^2 R}{R u_m}$ and $u_m = \frac{\int_0^{2\pi}\int_0^R urdrd\theta}{\pi R^2}$, which is called axial mainstream velocity. The following dimensionless parameters are introduced to make dimensionless the equation (16)

$\gamma = \left(\frac{\gamma_1 u_{HS} R}{q_w}\right), Br = \frac{\mu u_{HS}}{h q_w}, s_J = \frac{\sigma_e E_x^2 R}{q_w}$.

Where γ is the ratio of the heat generated by the interaction of the electric and magnetic fields to heat conduction, Br is the Brinkman number which describes the ratio of heat produced by viscous dissipation and heat transport by molecular conduction, S_J is the joule heating due to heat conduction. Then by using (18) the non-dimensional form of the equation (16) is obtained as

$$\left\{\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^* \frac{\partial T^*}{\partial r^*}\right)\right\} = \frac{k_f}{k_{eff}}(\gamma u^* - s_J) \qquad (20)$$

The corresponding non-dimensional boundary conditions are expressed as

$$T^*(-1) = 0, \ \frac{dT^*(0)}{dy^*} = 0 \qquad (21)$$

The analytical solution of the temperature distribution is derived and written as

$$T^* = F(r^*) - F(1) \qquad (22)$$

$F(1)$ is the functional value of the function $F(r^*)$ at $r^* = 1$. where.

$$F(r^*) = \int \frac{1}{r^*}\left(\int \frac{k_f}{k_{eff}}(\gamma u^* - s_J) r^* dr^*\right) dr^* \tag{23}$$

According to the obtained temperature distribution and velocity distribution, the non-dimensional bulk temperature can be written by the following expression:

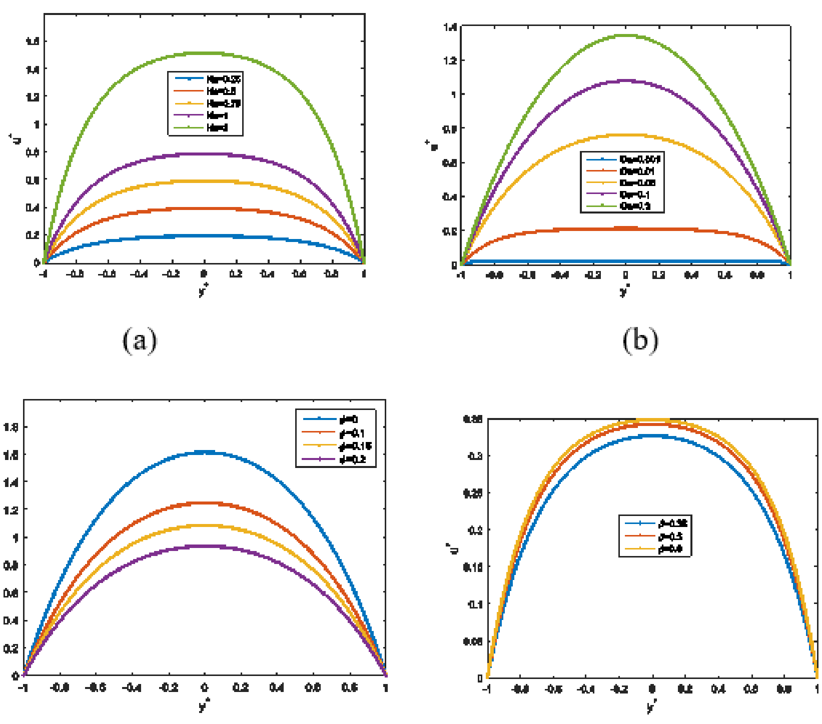
$$\widetilde{\theta^*} = \frac{\int_0^{2\pi}\int_0^1 u^* T^* r^* dr^* d\theta}{\int_0^{2\pi}\int_0^1 u^* r^* dr^* d\theta} = \frac{k_{eff}(T_w - T_m)}{R q_w} \tag{24}$$

In thermal transport phenomenon an important heat transfer parameter can be expressed as Nusselt number Nu, which illustrates the rate of heat transfer and can be defined as:

$$Nu = \frac{2Rq_w}{k_{eff}(T_w - T_m)} = -\frac{k_f}{k_{eff}}\frac{2}{\widetilde{\theta^*}} \tag{25}$$

3. RESULTS AND DISCOUSSION

The obtained results of the velocity and temperature distribution are discussed in this section. Further, the geometrical interpretation of streaming potential and rate of heat transfer are also illustrated.



(a) (b)

(c) (d)

Figure 2: Velocity distribution plots with variation of (a)Hartmann number (b)Darcy number (c) nano-particle volume friction (d) casson parameter.

The velocity and temperature profiles are represented in figure 2 and 3. It can be seen that the velocity distribution have the parabolic velocity profile and the velocity is maximum in the middle layers of the flow in the channel. When $S = 50$, the velocity profile have the increasing trend with the increases of the applied magnetic field and the increases of the Darcy number which are illustrated in figure 2(a) and 2(b) respectively. The velocity is slow when the Darcy number is very low. But in the presence of transverse electric field the velocity profile is depreciated with the increase values of the nano particle volume friction which is shown in figure 2(c). It can be demonstrated from the figure 2(c) that when $\phi = 0$, then the velocity is maximum and the velocity becomes slow for large value of nano particle volume friction parameter. Also it can be observed from figure 2(d) that the velocity has increasing trend with the increases of casson parameter.

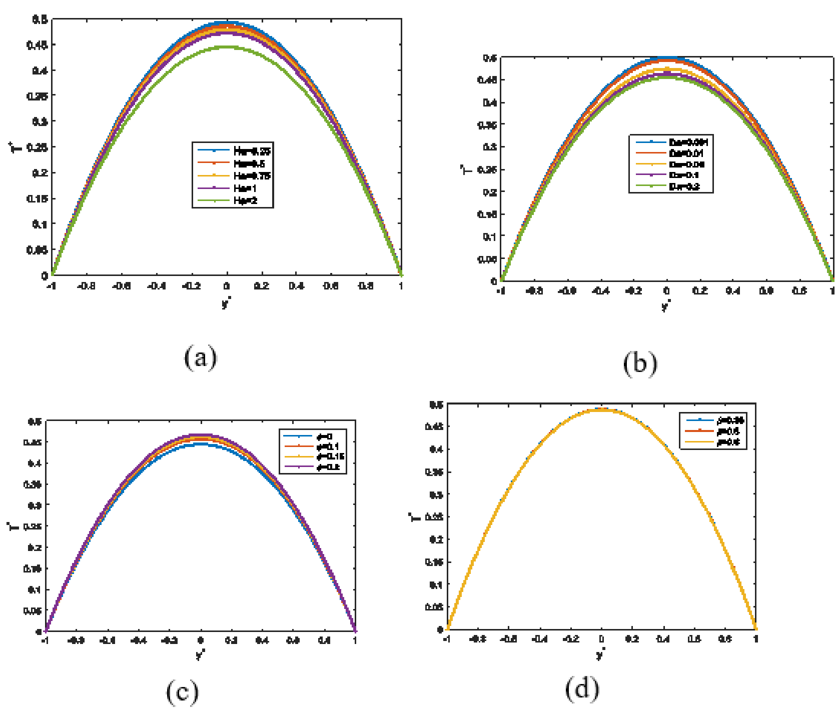
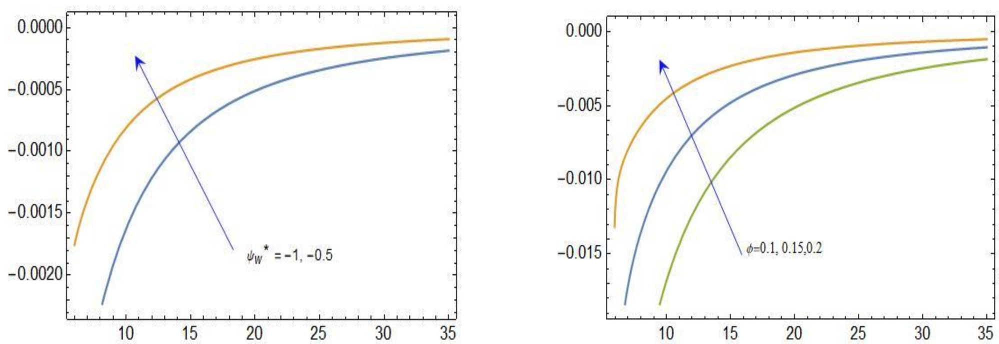


Figure 3: Temperature distribution plots with variation of (a)Hartmann number (b)Darcy number (c) nano-particle volume friction (d) casson parameter.

The geometrical approach of the temperature distribution is delineated in figure 3. It is demonstrated in figure 3(a) that the temperature profile decreaseded with the incrasing trend of the applied magnetic field. Also, when the Darcy number is increasing then the temperature is decreasing which is displayed in the figure 3(b). Figure 3(c) delineated that the temperature profile is decreased with increases of the nano particle volume friction. It is failed to trigger any apprehensible change in the associated temperature profile with the increasing trend of the casson parameter which is represented in figure 3(d). The temperature is maximum at the middle layers of the flow.



(a) (b)

Figure 4: Streaming potential (y-axis) plot with ω (x-axis) (a) with different wall zeta potential (b) with different values of the nano-particle parameter.

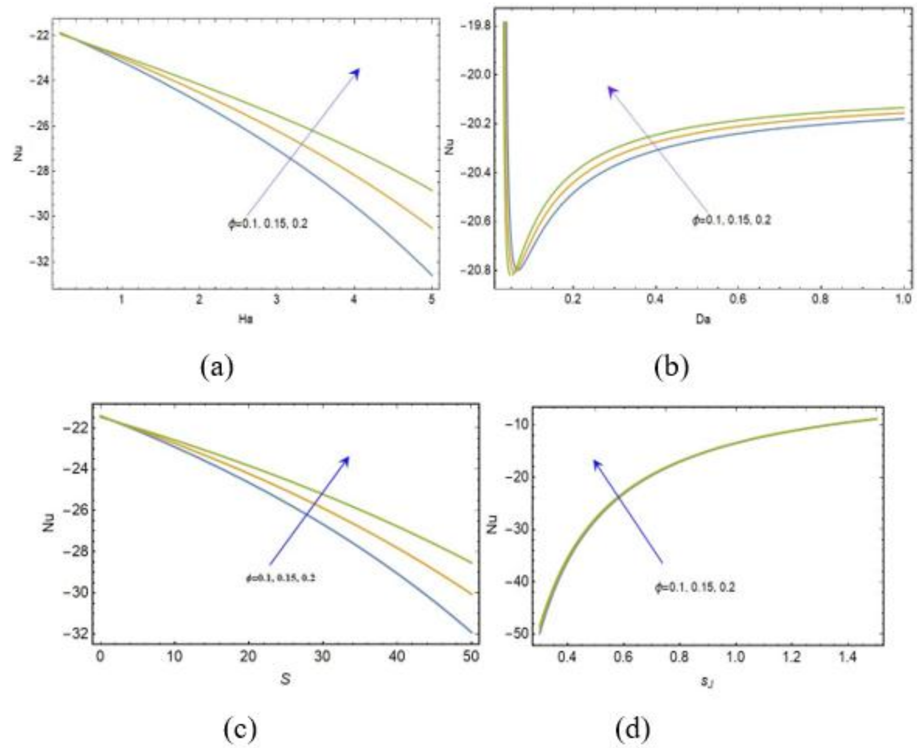


Figure 5: Effects of nano particle volume friction on Nusselt number with (a) Hartmann number, (b) Darcy number, (c)varying magnitude of Transverse electric field, (d)Joule heating parameter.

The Variation of Nusselt number with other pertinent parameters are displayed in figure 5. The Nusselt number is decreased gradually with the enhancement of the applied magnetic field and increases slowly for the higher value of the volume friction parameter of the nano-particles which is depicted in figure 5(a). From the figure 5(b), it is depicted that the Nusselt number is decreased swiftly for lower Darcy number but after a certain limit it is increased gradually with the increases of the Darcy number. However, for the higher value of the nano-particle parameter, the rate of heat transfer increases slowly. It may note from the figure 5(c) that the Nusselt number is decreased gradually with the enhancement of the transverse electric field and increases slowly for the higher value of the volume friction parameter of the nano-particles. The Nusselt number increases rapidly for an increasing trend of the joule heating parameter within the low range and the impact of the nano particle volume friction parameter on the Nusselt number are not very comprehensive with joule heating parameter which is illustrated in figure 5(d).

4. CONCLUSIONS

In this present study, the velocity and temperature distribution is analysed numerically for the EMHD flow of casson nano fluid through a porous microchannel under the streaming potential effects. The following observations can be drawn from this numerical study:

- The velocity is proportional to the nano particle volume friction but the temperature profile is inversely proportional to the nano particle volume friction.
- The velocity and temperature both are always maximum in the middle layers of the flow.
- The streaming potential is inversely proportional to the dimensionless EDL thickness parameter.
- The Nusselt number has an improvement with the increases of the volume friction parameter with the other pertinent parameter.

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