Pseudo-Spectral Solution of Thermal Instability of Nano-Fluid Flow through a Channel

Harsha S. V.[a] and Chandra Shekara G[b]

[a]School of Engineering and Technology, Jain University, Ramanaga,India

[b]BMS College of Engineering, Banaglore, India

ABSTRACT

A linear theory thermal instability of nano fluid through a rectangular channel bounded by rigid boundaries is investigated. The instability equations of nano fluid through the channel are derived by using boussinesq approximation and Buongiorno's model. These equations are discretized using non uniform grid points given by Gauss-Lobatto and then solved by using Chebyshev spectral collocation method with Chebyshev polynomials as basis of the solution. The generalized eigen values in terms of non-dimensional parameter such as modified Rayleigh number are obtained as function of wave number using QZ algorithm. The results display that the presence of nano particles suppresses the flow instability, but cannot absolutely get rid of it. As nano particle mass loading is increased, the region of unstable wave numbers is condensed from that of the pure Newtonian flow and the largest growth rate that governs the flow instability is reduced. The critical Rayleigh numbers increase and the unstable regions of small perturbations decrease, along with a decrease in the largest growth rates that govern the flow instability, therefore reinforcing the flow stability. Larger particles reduce the peak value of the velocity disturbance and hence attenuate the flow instability.

Keywords: Buongiorno's model, Cunnigham slip, spectral collocation, Chebyshev polynomials, Gauss-Lobatto points, QZ algorithm linear stability.

1. INTRODUCTION

The natural convection of fluid flow through rectangular channels arises in a large number of fields such as natural sciences, industries, engineering and technology. These consist of geothermal operations, petroleum products prosessing, thermal insulation and in the devise of solid-matrix heat exchangers, insulating materials, compact heat exchangers, and many others. Because of these widespread applications, there has been aextensive development in the field of natural convection during recent years. The improvements which have been taken place in this field over the years are well acknowledged in the literature: see, for example, Nishioka(1975),E. S. Asmolov and S. V. Manuilovich(2009),Jianzhong Lin (2014), Lennon O Naraigh (2013), E. A. Chinnova and O. A. Kabova(2011).

The stability of nano-fluid flow through bounded channels is in itself a fundamental topic of research due to its wide range of applications in cooling of electronic circuits, rheology and in enhancement of heat transfer. In particular, with the advent of nano materials there has been a significant increase in attention in the study of stability of nanofluid flows through rectangular channels bounded by a parallel plates in recent years.The result of thisthrows light relating to convective transport in nanofluids by Buongiorno (2006). E. S. Asmolov and S. V. Manuilovich (2009) had studied the stability of a horizontal plane-channel flow of a dilute suspension is theoretically and shown that the action of the sedimenting particles on the flow stability parameters is equivalent to the effect of a distributed flow stratification. J C Umavathi(2013) had found that the critical thermal Rayleigh number can be originate reduced or decreased by a substantial amount, depending on whether the basic nanoparticle distribution is top-heavy or bottom-heavy. The stability of flow of an incompressible nano fluid through a plane-parallel channe has been discussed on the origin of ancorrespondence with a hydrodynamic problem by JianzhongLin et. al(2014).

Nonetheless, studies pertaining to the stability of natural convection in a horizontal nano fluid layer under the impact of nanopartical density have been almost completely neglected despite its relevance and importance in a number of processes that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plates in a bath, exotic lubricants and colloidal fluids, liquids containing long-chain molecules as polymeric suspensions, electro-rheological fluids and so on. In view of the above observed phenomena, it is essential to explore the stability of natural convection of a nano fluid flow through horizontal channel with the plates are maintained at constant but dissimilar temperatures. The generalized eigenvalue problem is developed and it is solved numerically using the Chebyshev collocation method.

2. MATHEMATICA FORMULATION

The physical design of the problem is illustrated schematically in Fig.1. We consider a nano fluid flow through a horizontal layer bounded by rigid parallel paltes of width $2h$. The horizontal plates are maintained at constant temperature.A Cartesian coordinate system (x, y)is chosen with the origin in the middle of the horizontal layer, where the y -axis is taken perpendicular to the plates and the x -axis is vertically upwards, opposite in the direction of gravity. The plate at $y = -h$ is maintained at fixed temperature T_1 and while the plate at $y = h$ is maintained at fixed temperature $T_2(> T_1)$. The relevant basic equations under the Boussinesq approximation and following Buongiorno (2006)are given below:

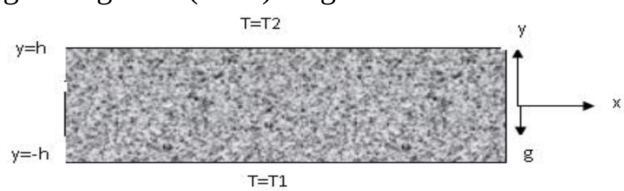

Fig. 1 Physical configuration.

Mass conservation Equation

$$\nabla \vec{q} = 0 \tag{1}$$

Momentum Equations

$$\rho_f \left[\frac{\partial \vec{Q}}{\partial t} + (\vec{Q}.\nabla)\vec{Q} \right] = -\nabla p + \mu \nabla^2 \vec{Q} - \frac{3\pi\mu dN}{C_c}(\vec{Q} - \vec{q}) + \rho \vec{g} \tag{2}$$

$$\rho_p \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} \right] = -\nabla p - \frac{18\mu}{d^2 C_c}(\vec{Q} - \vec{q}) \tag{3}$$

Conservation of Nano Particles

$$\frac{\partial n}{\partial t} + \nabla(n\vec{q}) = 0 \tag{4}$$

Energy Equation with equation of state

$$\frac{\partial T}{\partial t} + (\vec{Q}.\nabla)T = k\nabla T \tag{5}$$

$$\rho = \rho_{\cdot}\left[1 - \alpha\left(T - T_{\cdot}\right)\right] \tag{6}$$

The physical configuration considered in the present study leads to the following boundary conditions:

$$\vec{q} = \vec{Q} = 0 \text{ at } y = \pm h \tag{7}$$

$$T = T_1 \text{ at } y = -h \text{ and } T = T_2 \text{ at } y = h \tag{8}$$

Further to develop the eigen value problem we consider the initially the fluid flow is laminar, fully developed and unidirectional, thus we have the basic state in the form

$$\vec{U} = \vec{V} = U_b(\mathrm{y})\,\hat{\mathrm{j}},\ P = P_b(x, \mathrm{y}),\ T = T_b(y),\ \rho = \rho_b(y),\ n = N_b = 1 \tag{9}$$

Here the subscript b denotes the basic state. Under this circumstance, the basic state solution is found to be $U_b = -0.0833 R_e P_e R_{ean}\left[\left(\nabla Ty + 3\left(T_2 + T_1\right)\right)\left(y^2 + 1\right)\right]$ and $T_b = 0.5\left(\nabla Ty + T_2 + T_1\right)$.

To understand the stability of the nano fluid flow through the horizontal channel the basic state is perturbed by using infinitesimally very small disturbances given by

$$U = U_0 + U'\left(u_x, u_y\right), V = V_0 + V'\left(v_x, v_y\right), P = P_b + P', n = N_b + n', T = T_n + T' \tag{10}$$

Further the equations (1) to (6) are made non-dimensional using the scales U_b for velocity of fluid and nanoparticals, h for the space variable, $h(U_b)^{-1}$ for time, $0.1666\pi d^3 N_b$ for nanopartical density, ∇T for temperature ρ_0 for fluid density and $\rho_f U_b^2 h^{-1}$ for pressure and by using the equation(10) and using thelinearstabilitytheory we get

$$\frac{\partial u_x}{\partial t} + U_b\frac{\partial u_x}{\partial x} + u_y\frac{\partial U_b}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{\mathrm{R}_e}\left[\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2}\right] - \frac{z}{s_t C_c}\mathrm{N}_b(u_x - v_x) + P_e R_{ean} T \tag{11}$$

$$\frac{\partial u_y}{\partial t} + U_b\frac{\partial u_y}{\partial x} = -\frac{\partial P}{\partial y} + \frac{1}{\mathrm{R}_e}\left[\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2}\right] - \frac{z}{s_t C_c} n(u_y - v_y) \tag{12}$$

$$\frac{\partial v_x}{\partial t} + U_b\frac{\partial v_x}{\partial x} + v_y\frac{\partial U_b}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{s_t C_c}(u_x - v_x) \tag{13}$$

$$\frac{\partial v_y}{\partial t} + U_b\frac{\partial v_y}{\partial x} = -\frac{\partial P}{\partial y} + \frac{1}{s_t C_c}(u_y - v_y) \tag{14}$$

$$\frac{\partial n}{\partial t} + U_b\frac{\partial n}{\partial x} - \frac{\partial n_b}{\partial y} v_y = 0 \tag{15}$$

$$\frac{\partial T}{\partial t} + U_b\frac{\partial T}{\partial x} - u_y\frac{\partial T_b}{\partial y} = k\nabla^2 T \tag{16}$$

Further eliminating pressure between the equations (11), (12) and (13), (14)and by using the stream function formulation in the form $u_x = \psi_y$; $u_y = -\psi_x$ and $v_x = \varphi_y$; $v_y = -\varphi_x$. we get

$$\left(\frac{\partial}{\partial t} + U_b\frac{\partial}{\partial x}\right)\nabla^2\psi - \frac{\partial\psi}{\partial x}\frac{\partial^2 U_b}{\partial y^2} - \frac{\partial U_b}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} = \frac{1}{\mathrm{R}_e}\nabla^4\psi - \frac{zn}{s_t C_c}\nabla^2(\psi - \varphi) + P_e R_{ean}\frac{\partial T}{\partial y} \tag{17}$$

$$\left(\frac{\partial}{\partial t} + U_b\frac{\partial}{\partial x}\right)\nabla^2\varphi - \frac{\partial\varphi}{\partial x}\frac{\partial^2 U_b}{\partial y^2} - \frac{\partial U_b}{\partial y}\frac{\partial^2\varphi}{\partial x\partial y} = \frac{1}{s_t C_c}\nabla^2(\psi - \varphi) \tag{18}$$

$$\frac{\partial n}{\partial t}+u_b\frac{\partial n}{\partial x}-\frac{\partial n_b}{\partial y}\frac{\partial \varphi}{\partial x}=0 \tag{19}$$

$$\frac{\partial T}{\partial t}+u_b\frac{\partial T}{\partial x}-\frac{\partial \psi}{\partial x}\frac{\partial T_b}{\partial y}=k\nabla^2 T \tag{20}$$

Using normal mode solutionin the eqs (17) to (20) in the form $(\psi,\varphi,n,T)=\{(\psi,\varphi,n,T)(y)\}e^{i\alpha(x-ct)}$, we get,

$$i\alpha\left[(U_b-c)\left(D^2-\alpha^2\right)-DU_bD-D^2U_b-\frac{1}{R_e}\left(D^2-\alpha^2\right)^2\right]\psi=P_eR_{ean}\frac{\partial T}{\partial y}-\frac{zn}{s_tC_c}(D^2-\alpha^2)(\psi-\varphi) \tag{21}$$

$$i\alpha\left[\left(D^2-\alpha^2\right)(U_b-c)-DU_bD-D^2U_b\right]\varphi=\frac{1}{s_tC_c}(D^2-\alpha^2)(\psi-\varphi) \tag{22}$$

$$i\alpha(U_b-c)n-Dn_bi\alpha\varphi=0 \tag{23}$$

$$i\alpha(U_b-c)T-i\alpha\psi DT_b=k(D^2-\alpha^2)T \tag{24}$$

This equations are associated with the following boundary conditions:

$$\psi=D\psi=\varphi=T=n=0\ \ at\ \ y=\pm 1 \tag{25}$$

3. NUMERICAL SOLUTION

Equations (21) - (24) together with the boundary conditions (25) constitute an eigenvalue problem. This resulting eigenvalue problem is solved numerically using Chebyshev collocation method. The k^{th} order Chebyshev polynomial is given by,

$$\xi_j(y)=\cos(jz),\ z=\cos^{-1}(y) \tag{26}$$

The Chebyshev collocation points are given by $y_j=\cos\left(\pi j/N\right),\ j=0(1)N$. Here, the right and left wall boundaries match up to $j=0$ and N , respectively. The field variables ψ , φ, n and T can be approximated in terms of Chebyshev polynomials as follows:

$$\psi(y)=\sum_{j=0}^{N}\xi_j(y)\psi_j,\ \varphi(y)=\sum_{j=0}^{N}\xi_j(y)\varphi_j,\ n(y)=\sum_{j=0}^{N}\xi_j(y)n_j,\ T(y)=\sum_{j=0}^{N}\xi_j(y)T_j \tag{27}$$

The governing equations (21) - (24) are discretized in terms of Chebyshev polynomials to get

$$\begin{bmatrix}A_{11}&A_{12}&A_{13}&A_{14}\\A_{21}&A_{22}&A_{23}&A_{24}\\A_{31}&A_{32}&A_{33}&A_{34}\\A_{41}&A_{42}&A_{43}&A_{44}\end{bmatrix}\begin{bmatrix}\psi\\\varphi\\n\\T\end{bmatrix}=\frac{1}{i\alpha}\begin{bmatrix}B_{11}&B_{12}&B_{13}&B_{14}\\B_{21}&B_{22}&B_{23}&B_{24}\\B_{31}&B_{32}&B_{33}&B_{34}\\B_{41}&B_{42}&B_{43}&B_{44}\end{bmatrix}\begin{bmatrix}\psi\\\varphi\\n\\T\end{bmatrix}$$ where

$A_{11}=A_{22}=\left(D^2-\alpha^2\right)(U_b-c)-DU_bD-D^2U_b$, $A_{1j}=0,\ for\ j=2,3,4$ $A_{21j}=0,\ j=1,3,4$ $A_{32}=-Dn_b$,

$A_{33}=A_{44}=(U_b-c),\ A_{31}=A_{34}=0$ $A_{41}=-DT_b$, $A_{42}=A_{43}=0,$ $B_{12}=z(s_tC_c)^{-1}\left(D^2-\alpha^2\right)$, $B_{13}=0,$

$B_{11}=R_e^{-1}\left(D^2-\alpha^2\right)^2-z(s_tC_c)^{-1}\left(D^2-\alpha^2\right)$, $B_{44}=\left(D^2-\alpha^2\right)$, $B_{14}=P_eR_{an}$, $B_{22}=-(s_tC_c)^{-1}\left(D^2-\alpha^2\right)$,

$B_{23}=B_{24}=0,\ B_{4j}=0,\ j=1,2,3$ $B_{21}=(s_tC_c)^{-1}\left(D^2-\alpha^2\right)$, $B_{3j}=0,\ j=1,2,3,4$.

In the above equations D is Chebyshevdifferention matrix as defined in the Motsaet. al. (2019) and Chandra Shekara(2019). The above system of linear algebraic equations can be written in the following compact matrix form:

$$AX = cBX \tag{28}$$

In general, $c = c_r + ic_i$ is the complex number with c_r being the phase velocity and c_i is the growth rate. Implimenting the boundary contions in above matrix system in the form:
$\psi_0 = \psi_N = D\psi_1 = D\psi_{N-1} = 0$, $\varphi_0 = \varphi_N = 0$, $T_0 = T_N = 0$, $n_0 = 0$.

4. RESULTS AND DISCUSSIONS

The numerical results are presented with the main objective of investigating the effect of nanopartical density and on the stability of natural convection in a horizontal nano fluid layer. The non-dimensional parameters involved in the present study are the modified Rayleigh number R_{ean}, Stokes number S_t, nano particle density ration, Cunnigham slip correction C_c, and Reynolds number R_e. The Chebyshev collocation method is employed to extract the eigenvalues and the convergence of the numerical method is tested for different sets of parametric values by varying the order of base polynomial *N* and the results so obtained are tabulated in Table 1. From the table it is observed that four digits point accuracy can be achieved by retaining 25 terms in Eq. (27). As the number of terms increased in Eq. (27), the results are found to remain consistent and accuracy improved up to 7 digits for $N = 40$. Solutions of up to 8th digit could be reached by taking 51 terms of the approximation in Chebyshev collocation method and hence the results are obtained for $N = 50$ and for a fixed value of $N_b = 1$.

Table 1: Convergence of the Chebyshev collocation method with $C_c = 1.5$, $P_e = 0.75$.

s_t	$z = 0$	$z = 50$	$z = 70$
	R_{ean}	R_{ean}	R_{ean}
1	40.738381	31.694852	23.3412548
5	261.299459	255.972191	253.596912
10	532.833851	525.163845	520.760672

The nano particle density ratio z and stokes number s_t are found to have no control on the basic flow. Nonetheless, the modified-Rayleigh number R_{ean} and Reynolds number R_e are influence the same. Figures 2(a) and 2(b), respectively, show the influence of R_{ean} and R_e on the basic velocity U_b . These figures indicate that the velocity profiles are anti-symmetric about the vertical line at $y = 0$; however, they are not precisely centro-symmetric about $y = \pm 1$. In other words, in the half region, the basic velocity is in one direction and in the other half it is in the opposite direction and it is zero at $y = 0$. Moreover, decrease in R_{ean} is to suppress the fluid flow as it amounts to increase in the R_e (Fig. 2(a)) and a similar trend is noticed with decreasing R_{ean} (Fig. 2(b)).

The neutral stability curves in the $(\mathrm{R}_{ean}, \alpha)$ - plane are displayed in Figs. 3(a, b), (c) and (d) for various values of z , s_t and R_{ean} , respectively. In these figures, the portion below each neutral stability curve corresponds to stable region and the region above corresponds to instability. It is seen that the neutral stability curves exhibit single but different minimum with respect to the wave number for various values of z , s_t and R_{ean} . From the Figs. 3(a) and (b) it is observed that the effect of increasing s_t is to increase the region of stability when the instability is via stationary mode (Fig. 3a), while an opposite trend is noticed with increasing s_t when the instability is via travelling-wave mode (Fig. 3(b)). Figure 3(c) exhibits that increasing z is to decrease the region of stability. Similar effect is observed on the stability of the system with increasing R_{ean} and the same is evident from Fig. 3(d).

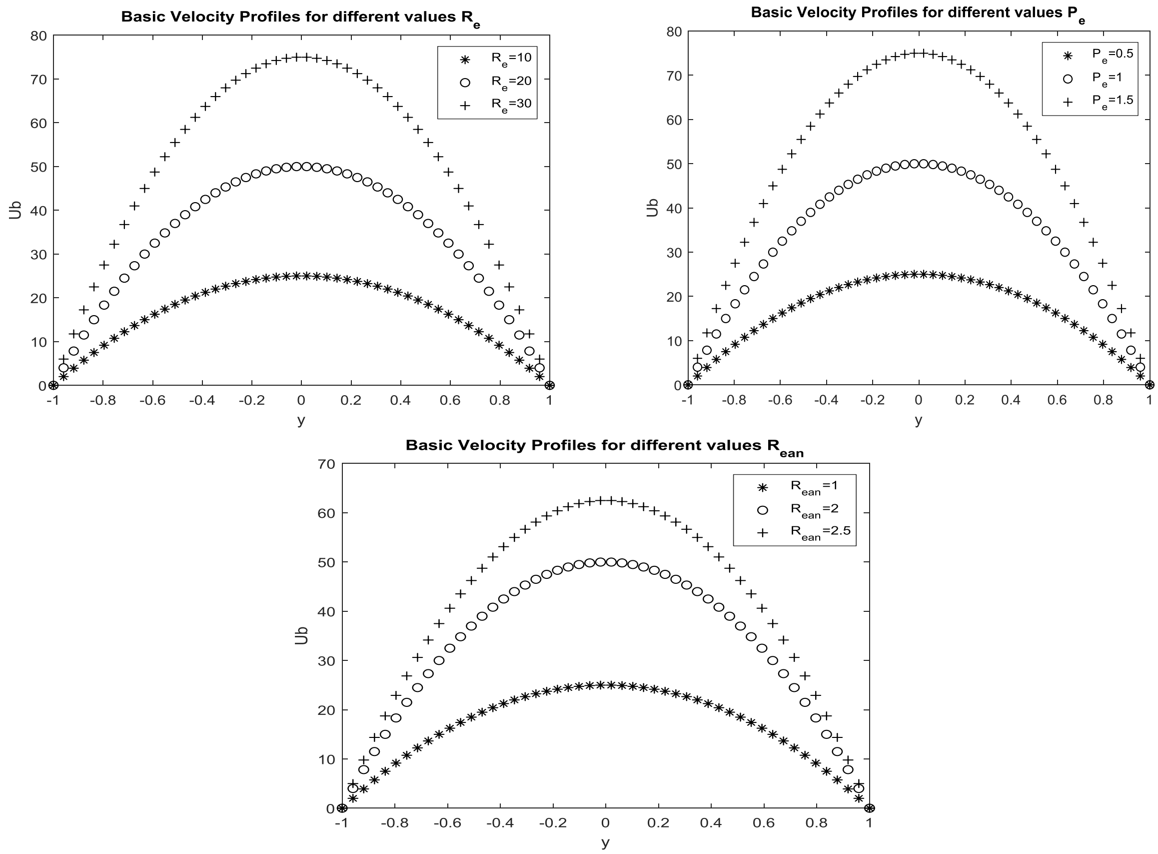

Fig. 2: Basic velocity profiles(a,b,c).

5. CONCLUSIONS

From the abovementioned study, it is observed that the nanopartical density has no influence on the basic velocity distribution. However, the effect of increasing R_{ean} is to introduce instability on the system but its effect is noted to be not so important. To the contrary, it exhibits a dual behavior once the instability is via travelling-wave mode. The value of s_t at which the transition from stationary to travelling-wave instability occurs and the wavelength of the critical wave remain invariant for all values of R_{ean}. Moreover, the value of z increases at which transition from stationary to travelling-wave instability occurs as the value of s_t increases. The streamlines and isotherms are also presented and found to mimic the behavior of stability curves observed before and after the change of mode of instability.

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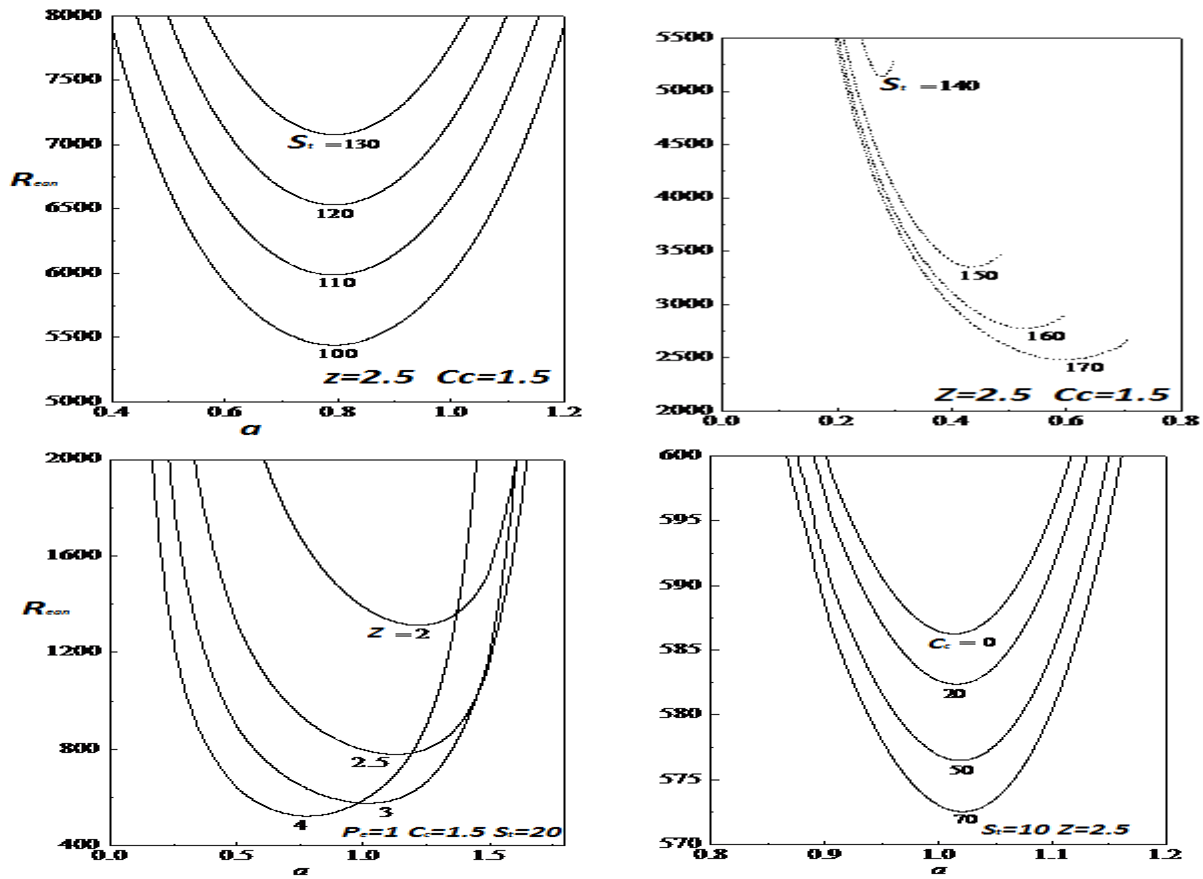

Fig. 3: Neutral stability curves. (a,c), Stationary modes (b), travelling-wave modes(d).

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