ROTORDYNAMIC ANALYSIS OF HIGH-SPEED ROTOR USED IN CRYOGENIC TURBOEXPANDER

Shantashree Jena¹, Partha Sarathi Kar², Debanshu S Khamari², Suraj K Behera²

¹Indira Gandhi Institute of Technology, Sarang, Odisha
²National Institute of Technology, Rourkela, Odisha
Mail id: beherask@nitrkl.ac.in

ABSTRACT
The size of advanced turbomachines such as turbocharger, turbo compressor, turbo-alternator, and turbo expander is decreasing to make it more compact. The decreasing size leads to high speed for better efficiency of the turbomachinery. The high-speed rotors are subjected to transverse vibration due to unbalance load and external radial load. The rotor dynamics study of such rotor is essential to estimate the level of vibration by predicting its critical speeds, mode shapes, and unbalanced response. In the present work the critical speeds, mode shapes, and unbalance response are predicted for a prototype rotor-bearing system used in cryogenic turboexpander. The designed rotational speed of the rotor is 2,40,000 rpm with radial unbalance near the bearing location is 40 mg-mm. This paper presents the rotordynamic analysis of the rotor using commercial finite element analysis software ANSYS®. The author believes the detailed finite element analysis will be quite useful of the researchers around the world.

Keywords: Finite Element Method, Turboexpander, Transverse vibration, Rotordynamics, ANSYS®, Critical speeds, Campbell Diagram, Unbalance response.

1. INTRODUCTION
Transverse vibration is one of the major causes of the failure of high-speed turbomachinery. This type of vibration creates whirling of the shaft which causes the shaft to bend considerably beyond certain spin speed and whirls around in this bent form. This leads to the instability of the system which further leads to failure of the rotor-bearing system. Rotor dynamics analysis helps to reduce such type of vibration by the detection of dynamic properties like critical speeds and mode shapes. Design of the rotor and bearing system must be done in such a way that the operating speed of turbomachinery is far away from the critical speed [1]. Likewise, mode shape is a definite pattern of vibration of the rotor at different natural frequencies which helps to determine the shape of the rotor at the critical speed also. Apart from critical speed and mode shapes unbalanced response are very important for the vibration analysis of the rotor. The unbalanced mass in a rotor which may occur due to material homogeneity, manufacturing process, and unsymmetrical slots, if present, causes excessive vibration in the rotating shaft. Thus the analysis of unbalance response is very much essential in high-speed rotor system to minimize these unbalance mass which can be done by proper rotor balancing procedure. For
these reasons, rotor dynamics analysis is an important procedure in the turbomachinery design process [2, 3].

Practiced methods of rotordynamics analysis can be done in the analytical approach, Transfer Matrix Method (TMM) and Finite Element Method (FEM). TMM approach is simple and quite useful for a simple rotor and disc models. FEM is the most practical approach, and it can be applied to a very large complex rotor-bearing system with its easy implementation using certain commercial software such as ANSYS®. Uses of this software also help to reduce computational effort and time. This paper targets to predict critical speeds, mode shapes and unbalance response of the prototype rotor using ANSYS® for the designed rotor-bearing system.

2. ROTOR DYNAMICS EQUATION
The general form of the equation of motion for all vibration problems is given by equation (1).

\[
[M] \ddot{q}(t) + [C] \dot{q}(t) + [K] q(t) = f(t)
\]  

(1)

Where,
- \([M]\) = Symmetric mass matrix
- \([C]\) = Symmetric damping matrix
- \([K]\) = Symmetric bearing or seal stiffness matrix
- \(q(t)\) = Generalized coordinates of the rotor in inertial coordinates
- \(f(t)\) = Force function i.e. unbalanced forces

The equation of motion in matrix form for an axially symmetric rotor rotating at constant spin speed (\(\Omega\)) is given by equation (2).

\[
[M] \ddot{q}(t) + ([C] + [G]) \dot{q}(t) + ([K] + [N]) q(t) = f(t)
\]  

(2)

Where,
- \([G]\) = Skew-symmetric gyroscopic matrix
- \([N]\) = Gyroscopic matrix of deflection for the inclusion of centrifugal elements

These gyroscopic matrices are greatly influenced by the rotational velocity \(\Omega\). The gyroscopic matrix \([G]\) contains inertial terms. When equation (2) is described in a rotating reference frame, Coriolis acceleration is also taken into consideration.
3. MODEL

The designed turboexpander rotor model is illustrated in Fig (1). The rotor model consists of a turbine, a compressor and a shaft with collar and bearings subsystem. The rotor is supported by a pair of journal (radial) bearings to support the radial load and a pair of thrust (axial) bearings to support the axial load. The designed speed of the rotating structure is taken as 2,40,000 RPM. The shaft material is K-Monel 500 and turbine and compressor are designed in Aluminum alloy. The properties of K-Monel 500 and Aluminum alloy are listed in Table-1.

![Turboexpander model and its schematic](image)

**Fig-1:** Turboexpander model and its schematic

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (in kg.m-3)</th>
<th>Young’s Modulus (in Pa)</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-Monel-500</td>
<td>7930</td>
<td>1.79E+11</td>
<td>0.32</td>
</tr>
<tr>
<td>Aluminum alloy</td>
<td>2770</td>
<td>7.1E+10</td>
<td>0.33</td>
</tr>
</tbody>
</table>

4. MESH GENERATION

Meshing is an important operation that has to be carried out properly before performing any analysis because the accuracy of the results depends on it. A trade-off is also essential between the quality of mesh and computational time needs to be maintained to keep the computational cost low. Different meshing methods that are used in ANSYS Workbench are Automatic Method, Tetrahedral Method, Hex Dominant Method, Sweep Method, and Multi-Zone Method [3]. To select which meshing is suitable for the model all the meshing methods are compared on the basis of the number of elements and number of nodes taking all the parameters related to mesh size same. The method with a larger number of nodes and a lesser
number of elements and comparatively lesser computational time is taken into consideration. From the analysis, the Automatic Meshing method is selected as it satisfies the above criteria. In automated meshing method, the meshing is done by varying sizing parameters such as relevance, element size for modal analysis. The no. of nodes, first natural frequency of free rotor and the computational time needed for each modal analysis are predicted and tabulated in Table-2 and the results are plotted in Fig (2) and Fig (3) showing the convergence plots. In both, the plots convergence is observed in the mesh sizing with the no. of nodes greater than 130346. Thus it can be determined that the mesh with a number of nodes greater than 130346 is a valid mesh and suitable for all analysis. The rotor model with proper meshing is shown in Fig (4) taking the above criteria and suitable analysis settings.

<table>
<thead>
<tr>
<th>No. of Nodes</th>
<th>First natural frequency (in Hz)</th>
<th>Computational time (in sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6498</td>
<td>1043.8</td>
<td>10</td>
</tr>
<tr>
<td>10096</td>
<td>1043.5</td>
<td>15</td>
</tr>
<tr>
<td>18589</td>
<td>1043.4</td>
<td>29</td>
</tr>
<tr>
<td>29265</td>
<td>1043.3</td>
<td>45</td>
</tr>
<tr>
<td>53214</td>
<td>1043.1</td>
<td>107</td>
</tr>
<tr>
<td>67494</td>
<td>1043.0</td>
<td>205</td>
</tr>
<tr>
<td>90931</td>
<td>1043.0</td>
<td>287</td>
</tr>
<tr>
<td>130346</td>
<td>1042.9</td>
<td>1044</td>
</tr>
<tr>
<td>160916</td>
<td>1042.9</td>
<td>2180</td>
</tr>
<tr>
<td>202286</td>
<td>1042.9</td>
<td>2657</td>
</tr>
</tbody>
</table>

**Fig-2:** Plot between No. of nodes and natural frequency
5. RESULTS AND DISCUSSION

Various analyses such as modal analysis and harmonic analysis are performed on the prototype-rotor bearing model to predict the critical speed and unbalance response.

5.1. MODAL ANALYSIS

Modal analysis was carried out in ANSYS® workbench taking a maximum number of modes as 10 with proper analysis settings. Total ten modes are calculated among which mode shape 2, 4, 6, 9 showing mode shapes at 1\textsuperscript{st}, 2\textsuperscript{nd}, 3\textsuperscript{rd} and 4\textsuperscript{th} critical speeds respectively. Mode shape 6 and 9 is also known as mode shape at 1\textsuperscript{st} and 2\textsuperscript{nd} bending critical speed respectively. Figs-(5 to 6) show the mode shapes at critical speeds with their damped natural frequencies.
Critical speeds are estimated using the Campbell diagram with designed rotational speed. Fig (7) shows the Campbell diagram. Critical speeds tabulated in Table (3) shows that speed within 2,40,000 RPM. The mode shape corresponds to the 1\textsuperscript{st} bending critical speed, or 3\textsuperscript{rd} critical speed has the appearance of half sine wave which is very important to predict to design of high-speed rotor.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Whirl direction</th>
<th>Mode stability</th>
<th>Critical speed(\text{rad/s})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>BW</td>
<td>STABLE</td>
<td>408.92</td>
</tr>
<tr>
<td>2.</td>
<td>BW</td>
<td>STABLE</td>
<td>6553.2</td>
</tr>
<tr>
<td>3.</td>
<td>FW</td>
<td>STABLE</td>
<td>6553.7</td>
</tr>
<tr>
<td>4.</td>
<td>BW</td>
<td>STABLE</td>
<td>13452</td>
</tr>
<tr>
<td>5.</td>
<td>FW</td>
<td>STABLE</td>
<td>14065</td>
</tr>
</tbody>
</table>
As the first two critical speeds are much less than the design speed i.e. 240,000 RPM, there is a chance of failure at the bearings near these speeds. Therefore, a preventive measure should be taken when the rotor undergoes near these critical speeds. This can be done by increasing mass flow rates of process gas during starting of the turboexpander.

5.2. HARMONIC RESPONSES
Harmonic analysis is carried out to show the frequency response of the system by applying an unbalance force of 40mg.mm in two radial bearing locations. The unbalance responses at the bearing near the expansion turbine side, the bearing near the brake compressor side, at the turbine end and the compressor end are shown in Figs (8-9) respectively.

From Fig-(8), it is seen that amplitudes at 1\textsuperscript{st} and 2\textsuperscript{nd} critical speeds are nearly 4µm and 2µm respectively. Similarly from the Fig-(9), the amplitudes at 1\textsuperscript{st} and 2\textsuperscript{nd} critical speeds near journal bearing on turbine side are nearly 4µm and 9µm respectively whereas in the compressor end amplitudes are nearly 4µm and 3µm for 1\textsuperscript{st} and 2\textsuperscript{nd} critical speed respectively. The designed radial clearance near journal bearings is nearly 25µm, so from above studies, the designed rotor was found to suitable to run at the designed speed of 2,40,000 rpm.
6. CONCLUSION

The objective of this paper was to analyze the behavior of high-speed rotor under designed rotational speed. Finite Element Method was used for the rotor dynamics analysis to predict the dynamic behavior of the rotor. From the modal and harmonic analysis, it was seen that at the turboexpander model is safe to operate at its designed rotational speed, i.e. 2,40,000 RPM. Thus it can be concluded as ANSYS® was the efficient and easy-to-handle software for the dynamic analysis of high-speed rotor model using Finite Element Method even with the much more complex geometry.

REFERENCES