Stochastic meshless method for Euler-Bernoulli beams with spatially varying random Young's modulus

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ABSTRACT

The current study proposes a stochastic element free-Galerkin method for the bending analysis of Euler-Bernoulli beam. Uncertainty in Young's modulus is modeled by considering it as a homogenous Gaussian random field. Discretization of random field is performed by shape function method. Perturbation method is used to study the first and second order moment statistics of deflection and rotation of beams. Beams with different boundary conditions are solved and the results have been validated with those obtained from a Monte Carlo simulation.

Keywords: Meshless method, Element free-Galerkin method, random field discretization, First and second-order perturbation method, Monte Carlo simulation

1. INTRODUCTION

Engineering structures are subjected to high degree of uncertainties during their lifetime. These uncertainties are associated with material properties, geometric properties, external loads, boundary conditions, etc. Therefore, a stochastic analysis of structures is necessary to improve the knowledge on uncertainties associated with the response. For such analysis, stochastic finite element method (SFEM) is widely used in literature [1,2]. However, mesh dependency in finite element method (FEM) can lead to issues in mapping and reduction in accuracy of the analysis results [3]. Meshless methods [3] are considered as acceptable alternatives in this respect [4,5,6]. Among different meshless methods available [3], the present study uses element free-Galerkin method (EFGM) [7,8] as the numerical tool due to its simplicity and comparability with FEM. EFGM is introduced in stochastic numerical analysis by Rahman and Rao [4]. Further, Arun et al. [5] used stochastic EFGM for the analysis of structural components with random initial damage. Later, Gupta and Arun [6] extended this method for the elastic buckling analysis of columns. However, bending analysis of Euler-Bernoulli beams using stochastic EFGM is not explored, and the current study investigates the same.

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2. EFGM FORMULATION FOR BENDING OF EULER-BERNOULLI BEAM

EFGM is used as a numerical tool for the deterministic analysis of Euler-Bernoulli beam. It uses moving least square (MLS) [9] method for the construction of shape functions and Galerkin weak form to develop the discretized system of equations [3,7,8].

2.1 MLS shape functions

The displacement of the problem domain is approximated as

$$\boldsymbol{v}^h(x) = \sum_{j=1}^{m} p_j(x)a_j(x) = \boldsymbol{p}^T(x)\boldsymbol{a}(x) \tag{1}$$

where, $\boldsymbol{p}(x)$ is the vector of basis functions of order m and $\boldsymbol{a}(x)$ is the vector of unknown coefficients [7].

Let the problem domain is discretized into n number of nodes. The nodal deflections and rotations are represented as $\boldsymbol{v} = [\hat{v}_1 \quad \hat{v}_2 \quad \ldots \quad \ldots \quad \hat{v}_n]^{\mathrm{T}}$ and $\boldsymbol{\theta} = [\hat{\theta}_1 \quad \hat{\theta}_2 \quad \ldots \quad \ldots \quad \hat{\theta}_n]^{\mathrm{T}}$ respectively.

The coefficient vector $\boldsymbol{a}(\mathbf{x})$ is obtained by minimizing the error norm given by

$$\mathrm{J}(x) = \sum_{i=1}^{n} \mathbf{w}(x - x_i)[(\boldsymbol{p}^{\mathbf{T}}(x_i)\boldsymbol{a}(x) - \hat{v}_i)^2 + (\boldsymbol{p}_x{}^{\mathbf{T}}(x_i)\boldsymbol{a}(x) - \hat{\theta}_i)^2] \tag{2}$$

where, $\mathbf{w}(\mathbf{x} - \mathbf{x_i})$ denotes the weight function at i^{th} node. The weight function should be nonzero inside the support domain and zero outside the domain [7]. $\boldsymbol{p}_{\mathbf{x}}(\mathbf{x})$ denotes the first derivative of the basis function with respect to $\mathbf{x}$. Minimizing the error norm gives:

$$\mathbf{A}(x)\mathbf{a}(x) = \mathbf{C_1}(x)\hat{\boldsymbol{v}} + \mathbf{C_2}(x)\hat{\boldsymbol{\theta}} \tag{3}$$

where $\mathbf{A}(x)$ is the moment matrix and $\mathbf{W}$ is the matrix containing weight functions at the point of interest.

$$\mathbf{A}(\mathbf{x}) = \boldsymbol{p}^{\mathbf{T}}\mathbf{W}\boldsymbol{p} + \boldsymbol{p}_x{}^{\mathbf{T}}\mathbf{W}\boldsymbol{p}_x \tag{4}$$

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}(x-x_1) & 0 & \ldots & 0 \\ 0 & \mathbf{w}(x-x_2) & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \mathbf{w}(x-x_n) \end{bmatrix} \tag{5}$$

Basis function $\mathbf{P}^{\mathbf{T}}$ and its first derivative with respect to x, $\mathbf{P}_x{}^{\mathbf{T}}$ are given by

$$\left.\begin{aligned} \boldsymbol{P}^{\mathrm{T}} &= [p^{\mathrm{T}}(\boldsymbol{x}_1) \quad p^{\mathrm{T}}(\boldsymbol{x}_2) \dots\dots\dots p^{\mathrm{T}}(\boldsymbol{x}_n)] \\ \boldsymbol{P}_x{}^{\mathrm{T}} &= [p_x{}^{\mathrm{T}}(\boldsymbol{x}_1) \quad p_{\mathbf{x}}{}^{\mathrm{T}}(\boldsymbol{x}_2) \dots\dots\dots p_{\mathbf{x}}{}^{\mathrm{T}}(\boldsymbol{x}_n)] \end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned} \mathbf{C_1}(x) &= \boldsymbol{P}^{\mathbf{T}}\mathbf{W} \\ \mathbf{C_2}(x) &= \boldsymbol{P}_x{}^{\mathbf{T}}\mathbf{W} \end{aligned}\right\} \tag{7}$$

and the coefficient vector is given by

$$\boldsymbol{a}(x) = \mathbf{A^{-1}}(x)\mathbf{C_1}(x)\widehat{\boldsymbol{v}} + \mathbf{A^{-1}}(\mathbf{x})\mathbf{C_2}(x)\widehat{\boldsymbol{\theta}} \tag{8}$$

Finally, the equation for displacement is obtained as

$$\boldsymbol{v}(x) = \sum_{i=1}^{n} \boldsymbol{\varphi}_{\mathbf{v}}{}^{\mathrm{T}}\widehat{\boldsymbol{v}} + \sum_{i=1}^{n} \boldsymbol{\varphi}_{\boldsymbol{\theta}}{}^{\mathrm{T}}\widehat{\boldsymbol{\theta}} \tag{9}$$

where, φ_v and φ_θ are the MLS shape functions for deflection and rotation respectively [6, 11].

$$\left.\begin{aligned} \boldsymbol{\varphi}_{\mathbf{v}} &= \boldsymbol{p}^{\mathbf{T}}\mathbf{A^{-1}C_1} \\ \boldsymbol{\varphi}_{\boldsymbol{\theta}} &= \boldsymbol{p}^{\mathbf{T}}\mathbf{A^{-1}C_2} \end{aligned}\right\} \tag{10}$$

Derivatives of these shape functions can be calculated as in the following equations with $(-)_{,x} = \frac{\partial(-)}{\partial x}$ and $(-)_{,xx} = \frac{\partial^2(-)}{\partial x^2}$

$$\left.\begin{aligned} \frac{\partial \boldsymbol{\varphi}_{\mathbf{v}}}{\partial x} &= \boldsymbol{p}_{,x}{}^{\mathbf{T}}\mathbf{A^{-1}C_1} + \boldsymbol{p}^{\boldsymbol{T}}\left(\mathbf{A^{-1}C}_{\mathbf{1},x} + \mathbf{A^{-1}}_{,x}\mathbf{C_1}\right), \\ \frac{\boldsymbol{\partial \varphi_\theta}}{\partial x} &= \boldsymbol{p}_{,x}{}^{\mathbf{T}}\mathbf{A^{-1}C_2} + \mathbf{p}^{\mathbf{T}}\left(\mathbf{A^{-1}C}_{\mathbf{2},x} + \mathbf{A^{-1}}_{,x}\mathbf{C_2}\right) \\ \frac{\partial^2 \boldsymbol{\varphi}_{\mathbf{v}}}{\partial x^2} &= \boldsymbol{p}_{,\mathbf{xx}}{}^{\mathbf{T}}\mathbf{A^{-1}C_1} + \mathbf{2}\boldsymbol{p}_{,x}{}^{\mathbf{T}}\left(\mathbf{A^{-1}C}_{\mathbf{1},x} + \mathbf{A^{-1}}_{,x}\mathbf{C_1}\right) + \boldsymbol{p}^{\mathbf{T}}\left(\mathbf{A^{-1}C}_{\mathbf{1},xx} + \mathbf{2A^{-1}}_{,x}\mathbf{C}_{\mathbf{1},x} + \mathbf{A^{-1}}_{,xx}\mathbf{C_1}\right) \\ \frac{\partial^2 \boldsymbol{\varphi}_{\boldsymbol{\theta}}}{\partial x^2} &= \boldsymbol{p}_{,xx}{}^{\mathbf{T}}\mathbf{A^{-1}C_2} + \mathbf{2}\boldsymbol{p}_{,x}{}^{\mathbf{T}}\left(\mathbf{A^{-1}C}_{\mathbf{2},x} + \mathbf{A^{-1}}_{,x}\mathbf{C_2}\right) + \boldsymbol{p}^{\mathbf{T}}(\mathbf{A^{-1}C}_{\mathbf{2},xx} + \mathbf{2A^{-1}}_{,x}\mathbf{C}_{\mathbf{2},x} + \mathbf{A^{-1}}_{,xx}\mathbf{C_2} \end{aligned}\right\} \tag{11}$$

where $\mathbf{A^{-1}}_{,x} = -\mathbf{A^{-1}A}_{,x}\mathbf{A^{-1}}$[7] and $\mathbf{A^{-1}}_{,xx} = -\mathbf{A^{-1}}_{,x}\mathbf{A}_{,x}\mathbf{A^{-1}} - \mathbf{A^{-1}A}_{,xx}\mathbf{A^{-1}} - \mathbf{A^{-1}A}_{,x}\mathbf{A^{-1}}_{,x}$ [6]. In order to obtain nonsingular moment matrices ($\mathbf{A}$), a sufficient number of nodes should be included in the domain of influence of point under consideration [6,7].

2.2 Formulation of the beam bending problem

An Euler Bernoulli beam is governed by the following differential equation

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(EI\left(\frac{\mathrm{d}^2 v}{\mathrm{d}x^2}\right)\right) + \mathrm{q}(\boldsymbol{x}) = 0 \ \text{ in } \Omega = (0, l) \tag{12}$$

where E is the Young's modulus, I is the moment of inertia, $\boldsymbol{v}$ is the transverse displacement, $\mathrm{q}(\boldsymbol{x})$ is the force acting on the beam, l is the length of the beam and Ω is the problem domain. Applying the essential and natural boundary conditions as well as constructing the weak form reduces the problem into a system of algebraic equations given by

$$\mathbf{F} = \mathbf{Kd} \tag{13}$$

where $\mathbf{K}$ is the stiffness matrix; $\mathrm{K}_{\mathbf{ij}} = \int_0^{\mathrm{L}} \mathbf{B}_{\boldsymbol{i}} EI\, \mathbf{B}_{\mathbf{j}}\, \mathrm{dx}$ and $\mathbf{B}_{\boldsymbol{i}}$ is the strain-displacement matrix for the i^{th} node; $\mathbf{B}_{\boldsymbol{i}} = \left[\frac{\partial^2 \boldsymbol{\varphi}_{vi}}{\partial x^2} \ \frac{\partial^2 \boldsymbol{\varphi}_{\theta i}}{\partial x^2}\right]$, $\mathbf{d} = [v_1 \quad \theta_1 \quad v_2 \quad \theta_2 \dots\dots\dots\dots v_n \quad \theta_n]$ and $\mathbf{F}$ is the force vector; $\mathbf{F} = \int_0^{l} \boldsymbol{\Phi}\ \mathrm{q}(x)\mathrm{dx}$ where, $\boldsymbol{\Phi}$ is the matrix of MLS shape functions. Scaled transformation method [10,11] is used for the enforcement of essential boundary conditions. Incorporation of randomness in the material property makes the problem a stochastic one and the following section discusses the formulation and solution of such problems using EFGM.

3. STOCHASTIC EFGM FORMULATION

The current study models Young's modulus (E) as a spatially varying random variable. It is considered as a Gaussian random field, $E(x) = \mu_{\mathrm{E}}(1 + \beta(x))$ with μ_E as the mean modulus of elasticity and $\beta(x)$ as the zero-mean Gaussian field [12]. Exponential auto-covariance kernel is assumed for $\beta(x)$, which is given by $\Gamma_\beta = \sigma_\beta{}^2 \exp\left[-\left(\frac{|\boldsymbol{\delta}\mathbf{x}|}{\mathrm{d}_x}\right)\right]$ where, σ_β is the coefficient of variation of $\mathrm{E}(x)$, $\delta\mathbf{x}$ is the distance between two points $\mathbf{x_1}$ and $\mathbf{x_2}$ in the problem domain and d_x is the correlation length parameter. σ_β can be expressed as $\sigma_\beta = \frac{\sigma_E}{\mu_E}$ where, σ_E is the standard deviation of $E(x)$ [6,12,13]. Incorporation of $E(x)$ into the governing differential equation makes the problem a stochastic one and hence the stiffness matrix also. However, $E(x)$ being a random field, explicit expression for the same is not available and needs to be approximated.

3.1.Random field discretization

Due to its simplicity and high degree of accuracy, the shape function (SF) method [2] is preferred for the approximation of the random field. The Gaussian random field $\beta(x)$ is discretized into N number of random variables and approximated as $\beta(x) = \sum_{i=1}^{N} \boldsymbol{\Phi}_{\mathrm{i}}(x)\beta_i$ [2] where, β_i are the nodal values of the random field which is a Gaussian random variable with zero mean and same covariance structure as that of $\beta(x)$, and $\boldsymbol{\Phi}_{\mathrm{i}}(x)$ is the MLS shape function.

The stochastic beam bending problem is analyzed by first order perturbation (FOP) and second order perturbation (SOP) methods and the same have been validated by Monte Carlo simulation (MCS).

3.2.Perturbation method

Euler-Bernoulli beam is solved for deflections and rotations using perturbation method [12] which assumes the variance of the random field to be small [5]. The set of equations for stochastic beam bending problem described in Eq. (4) can be written as

$$\mathbf{F} = \mathbf{K}(\boldsymbol{\beta})\,\mathbf{d}(\boldsymbol{\beta}) \tag{14}$$

In perturbation method, each term in the equation is expanded by Taylor's series approximation and coefficients of the same order terms on both sides are equated to determine the displacement and its derivatives. Taylor's series expansions for $\mathbf{K}$ and $\mathbf{d}$ about $\beta = 0$ are given by the following expressions.

$$\left.\begin{aligned} \mathbf{K}(\boldsymbol{\beta}) &= \mathbf{K}_0 + \sum_{i=1}^{N} \mathbf{K}_{,\mathrm{i}}\beta_i + \frac{1}{2}\sum_{i,j=1}^{N} \mathbf{K}_{,\mathrm{ij}}\beta_i\beta_j + \cdots \\ \mathbf{d}(\boldsymbol{\beta}) &= \mathbf{d}_0 + \sum_{i=1}^{\mathrm{N}} \mathbf{d}_{,\mathrm{i}}\beta_i + \frac{1}{2}\sum_{i,j=1}^{\mathrm{N}} \mathbf{d}_{,\mathrm{ij}}\beta_i\beta_j + \cdots \end{aligned}\right\} \tag{15}$$

where, $(-)_0 = (-)(0)$ and $(-)_{,i} = \frac{\partial(-)}{\partial\beta_i}$, $(-)_{,ij} = \frac{\partial^2(-)}{\partial\beta_i\partial\beta_j}$ which are evaluated at the mean value of β. Expressions for $\mathbf{d}_{\mathbf{0}}, \mathbf{d}_{,i}$ and $\mathbf{d}_{,ij}$ are obtained as follows.

$$\left.\begin{aligned} \mathbf{d}_{\mathbf{0}} &= (\mathbf{K_0})^{-1}\ \mathbf{F_0} \\ \mathbf{d}_{,\mathrm{i}} &= (\mathbf{K_0})^{-1}\left(-\mathbf{K}_{,\mathrm{i}}\mathbf{d}_0\right) \\ \mathbf{d}_{,ij} &= (\mathbf{K_0})^{-1}\left(-(\mathbf{K}_{,i}\mathbf{d}_{,j} + \mathbf{K}_{,j}\mathbf{d}_{,i}))\right) \end{aligned}\right\} \tag{16}$$

Now, expectation and variance operators are taken both sides of Eq. (14) in order to find the mean and standard deviation of displacements. Mean and variance for the FOP method are obtained as;

$$\left.\begin{aligned} \mu_{\mathbf{d}} &= \mathbf{d}_0 \\ \sigma_{\mathbf{d}}{}^2 &= \sum_{\mathrm{i,j=1}}^{\mathrm{N}} \mathbf{d}_{,\mathrm{i}}\mathbf{d}_{,\mathrm{j}}{}^{\mathrm{T}}\Gamma_\beta(\beta_{\mathrm{i}}\,,\ \beta_{\mathrm{j}}) \end{aligned}\right\} \tag{17}$$

Mean and variance for the SOP method can be written as;

$$\left.\begin{array}{c} \mu_{\mathbf{d}} = \mathbf{d}_0 + \frac{1}{2}\sum_{\mathrm{i,j=1}}^{\mathrm{N}} \mathbf{d}_{,ij}\ \Gamma_{\beta}(\beta_i\,,\,\beta_{\mathrm{j}}) \\ \sigma_{\mathbf{d}}{}^2 = \sum_{i,j=1}^{N} \mathbf{d}_{,i}\mathbf{d}_{,j}{}^{\mathrm{T}}\,\Gamma_{\beta}(\beta_{\mathrm{i}}\,,\,\beta_{\mathrm{j}}) + \frac{1}{4}\sum_{i,j,k,l=1}^{N} \mathbf{d}_{,ij}\mathbf{d}_{,k\mathrm{l}}{}^{\mathrm{T}}\{\Gamma_{\beta}(\beta_{\mathrm{i}}\,,\,\beta_l)\,\Gamma_{\beta}(\beta_{\mathrm{j}}\,,\,\beta_k) \\ +\,\Gamma_{\beta}(\beta_i\,,\,\beta_k)\,\Gamma_{\beta}(\beta_j\,,\,\beta_l)\} \end{array}\right\} \qquad (18)$$

Eq. (17) and Eq. (18) are employed to determine the first and second order moment statistics (mean and variance) of deflection and rotation of the Euler-Bernoulli beam. These results are then validated using conventional MCS by taking 5000 number of simulations [6].

4. NUMERICAL EXAMPLES

Material of the beam is chosen as mild steel with $E = 2.1\,\mathrm{x}10^{11}\ \mathrm{N/m^2}$. Length of the beam ($l$) is taken as 1m and cross-section is circular with diameter 0.02m. Beams with following boundary conditions are solved.

Case 1: Cantilever beam with uniformly distributed load (*udl*) over the whole span

Case 2: Simply supported beam with *udl* over the whole span

Case 3: Beam fixed at both ends and with *udl* over the whole span

Case 4: Propped cantilever beam with *udl* over the whole span

Value of *udl* is taken as 1kN/m.

These four cases are analyzed with a deterministic EFGM formulation and the results are compared with the available analytical solutions. Stochastic analysis is done with perturbation method and the mean and standard deviation for each case are then validated with MCS. Details of deterministic analysis and stochastic analysis for all the cases are discussed here.

4.1. Deterministic analysis

Deterministic analysis of beams is carried out and a convergence study is done in order to determine the suitable type of weight function, number of nodes and the EFGM scaling parameter d_{max} . Among the different weight functions (exponential, cubic as well as quartic splines [3]) it is found that the results are acceptable with the use of exponential weight function.

The number of nodes is varied from 7 to 21 and in all the cases a node number of 11 gives convereged results with the following d_{max} values (Table 1).

Table 1. Results for deterministic analysis of beams

	Case 1	Case 2	Case 3	Case 4
d_{max}	2.5	2.5	2	1.96

In all the cases numerical integration is performed by two-point Gauss quadrature and a cubic polynomial basis function is used in the field variable approximation.

4.2.Stochastic analysis

Discretization of the random field is performed by shape function method which is incorporated in the numerical integration part where Young's modulus (E) is modeled as a homogenous Gaussian random field. The mean and standard deviation of displacements for each case of beams are determined using FOP and SOP. These results are then compared with the MCS.

Value of coefficient of variation of Young's modulus (σ_β) is taken as 0.1. Correlation length is fixed as 1. For simplicity, the number of discretization points for the random field is kept the same as the number of nodes used in EFGM [12].

4.2.1 Computational time

Perturbation method is compared with MCS in terms of computational time required. The normalized computational time required for stochastic analysis using both the perturbation method and MCS is listed in Table 2.

Table 2. Comparison of computational time of MCS and perturbation techniques

	Computational time (seconds)	
	MCS	Perturbation method (FOP and SOP)
Case 1	1	5.222×10^{-3}
Case 2	1	7.056×10^{-3}
Case 3	1	3.972×10^{-3}
Case 4	1	5.640×10^{-3}

Since the second derivative of the stiffness matrix is absent in the formulation, there is no distinction on the computational time requirement of FOP and SOP.

From Table 2, it is very clear that the perturbation technique gives faster (about 180 times) converged results compared to that of MCS in the case of Euler-Bernoulli beam.

4.2.2 Mean and standard deviation along the axis of the beam

The mean and standard deviation of deflection and rotation of beams with different boundary conditions are studied using FOP, SOP and MCS. Figure 1 shows the variation of the mean as well as the standard deviation of deflection and rotation of beams along the beam axis for case 1.

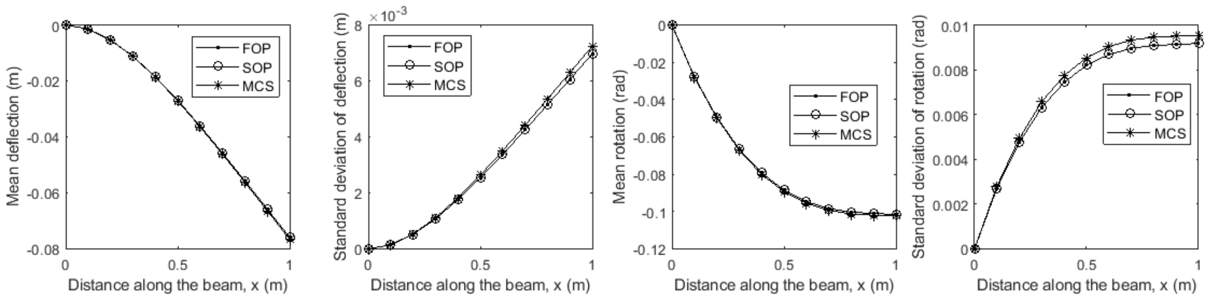

Figure 1. Variation of mean and standard deviation along the axis of the beam for case 1

From figure 1, it is seen that the mean deflection and mean rotation vary along the beam axis in a similar fashion as that of the deterministic analysis. It is also observed that the mean and the standard deviation of deflection and rotation determined using perturbation methods are comparable with those obtained from MCS. This variation of deflection and rotation are plotted for all the other cases also as shown in figure 2 to figure 4.

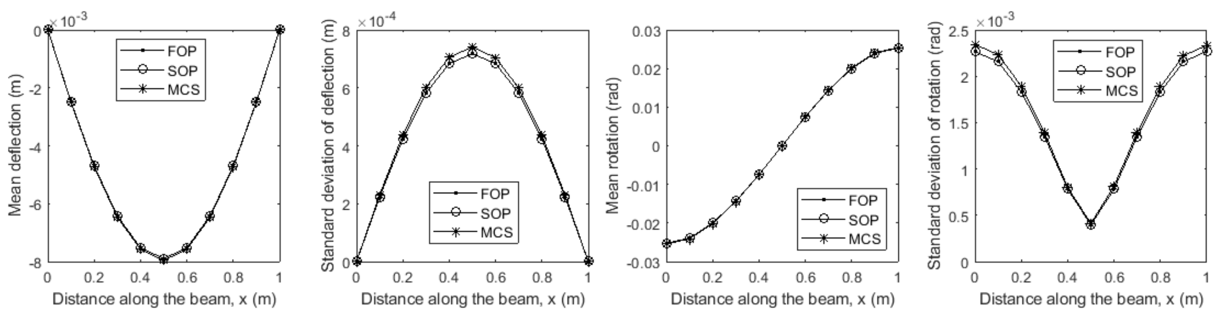

Figure 2. Variation of mean and standard deviation along the axis of the beam for case 2

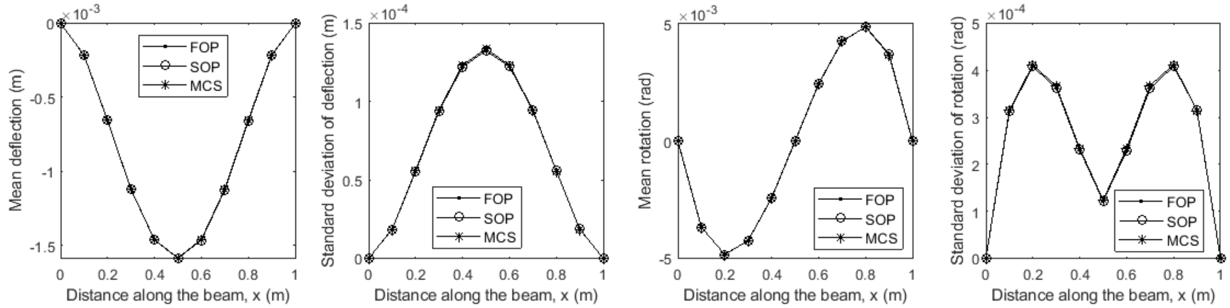

Figure 3. Variation of mean and standard deviation along the axis of the beam for case 3

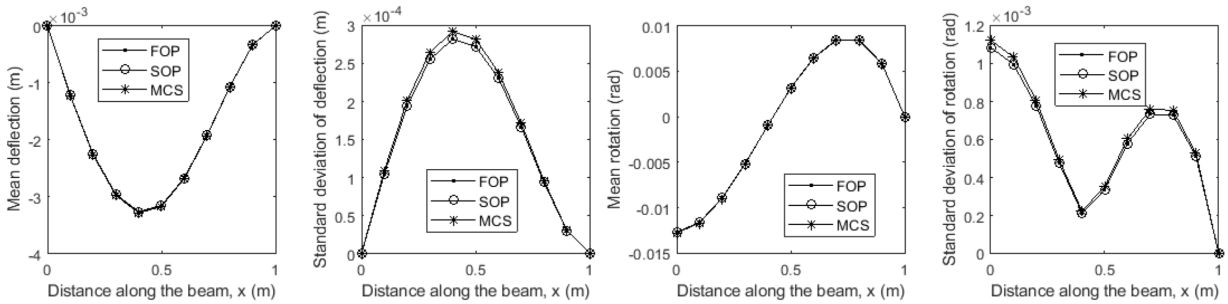

Figure 4. Variation of mean and standard deviation along the axis of the beam for case 4

From figure 2 to figure 4, it is found that the mean deflection and the mean rotation show a similar variation as that of the deterministic analysis for the particular boundary condition. The resulting mean and standard deviation of FOP and SOP are found in good agreement with those obtained from MCS and which proves the EFGM to be an effective tool for the stochastic analysis of Euler-Bernoulli beam.

A parametric study is conducted in order to study the effect of coefficient of variation and correlation length of the input random field on the mean and standard deviation of deflection and rotation of beams and the same have been discussed here.

4.2.3 Mean and standard deviation with coefficient of variation of random field

Keeping the correlation length as 1 and the number of random variables as 11, the coefficient of variation of the input random field is varied from 5% to 20%. For all the cases, variation of first and second moment statistics with coefficient of variation of random field is studied. Figure 5 shows this variation for case 1. A point which is located at $0.8m$ from the left end of the beam is considered for the study.

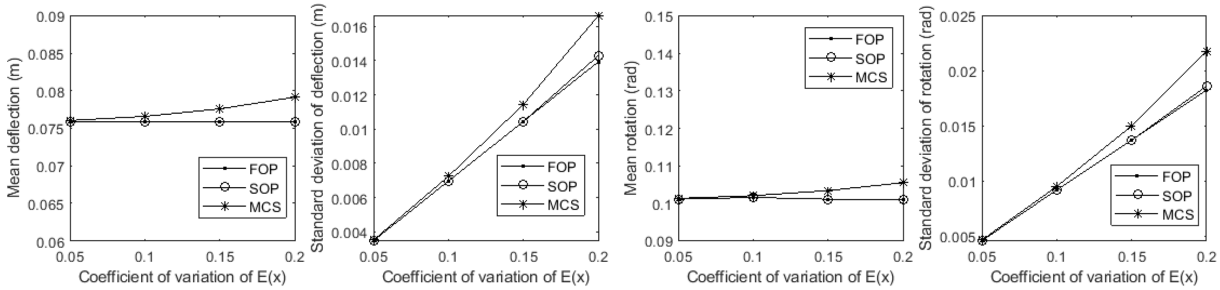

Figure 5. Variation of mean and standard deviation with coefficient of variation for case 1

From figure 5, it can be seen that the results of MCS and perturbation methods are comparable when the coefficient of variation of random field is within 10% and acceptable variation is observed till 15%. After this 15% of the coefficient of variation, perturbation result deviates away from those obtained from MCS. Since the second-order derivatives of the stiffness matrix and force matrix are absent in this particular problem, there is no variation in the mean and the standard deviation among the results of both FOP and SOP.

4.2.4 Mean and standard deviation with correlation length

For the coefficient of variation of 10% and the number of random variables 11, the correlation length is varied from 0.5 to 2. The variation of the mean and the standard deviation are plotted for case 1 as shown in figure 6. In this case also, the point under consideration is located at $0.8m$ from the left end of the beam.

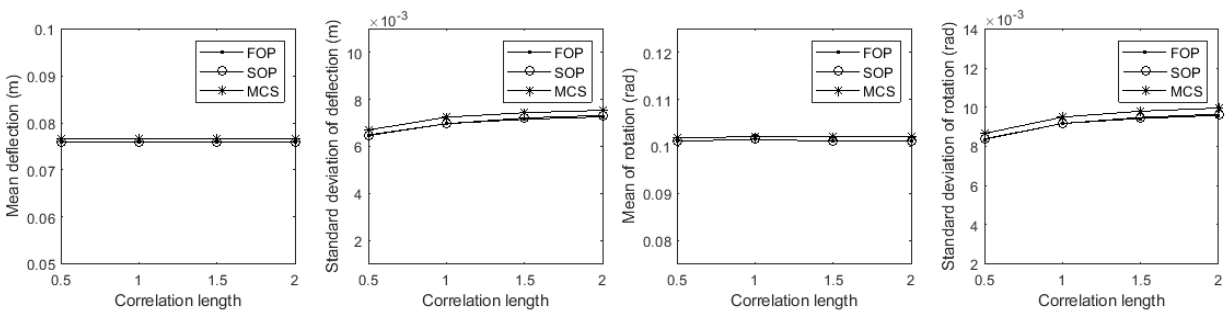

Figure 6. Variation of mean and standard deviation with correlation length for case 1

From figure 6, it can be observed that stochastic EFGM gives accurate results regardless of the correlation length of the input random field.

5. CONCLUSIONS

Bending analysis of the Euler-Bernoulli beam has been carried out using stochastic element-free Galerkin method. Randomness in material property is incorporated by treating Young's modulus of the material as a homogenous Gaussian random field. Shape function method is utilized to discretize the random field. Numerical examples of beams with four different boundary conditions are solved - cantilever beam, simply supported beam, fixed beam and propped cantilever beam.

A convergence study with deterministic analysis has been done and suitable type of weight function, EFGM scaling parameter (d_{max}), number of nodes etc. are selected for each type of boundary condition. First and second moment statistics of deflection as well as rotation are studied using first order and second order perturbation methods and the same have been validated using MCS. Results of perturbation method are found in good agreement with that of MCS.

Further, from the comparison of computational time, it is found that both the FOP and SOP are around 180 times faster than MCS. Thus it makes EFGM an efficient tool for stochastic analysis of beams. Parametric study is conducted in order to find the effect of coefficient of variation of random field and correlation length of the same on the mean and standard deviation of deflection and rotation of beams. It is found that the stochastic EFGM gives better results of first and second moment statistics when the coefficient of variation of random field is less than 0.15 whereas the results are not affected by the correlation length of the input random field.

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