Section Code: SM2 (Computational Solid Mechanics)

# Coupled Thermomechanical Analysis of Shape Memory Alloy Beam Actuator Considering Large Deformation

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## ABSTRACT

In this article, the coupled thermomechanical responses of shape memory alloy (SMA) strip actuator in the shape adaptive structures are investigated. The thermodynamical constitutive model of Qidwai and Lagoudas (2000) is implemented in non-linear finite element framework to analyze the unique characteristics of a SMA strip. The Green-Lagrange strain-displacement relationships and Second Piola-Kirchhoff stress measure are considered to account for the effect of large displacements, large strains, and material nonlinearity. For the numerical results presented in this article, the open-source finite element library, deal.II, has been utilized. To account for the effect of exothermic or endothermic nature of transformation, the stress and thermal equilibrium equations are solve simultaneously considering material level coupling terms, i.e., latent heat and thermoelastic heat. At each time step, the increment in solution is obtained, considering the material algorithm, following Newton-Raphson iterative scheme. The results highlight a significant difference in the transient response of SMA structures while thermal coupling terms are considered; illustrating the importance of the coupling phenomenon in the analysis of SMA based shape adaptive structures.

**Keywords:** Shape Memory Alloy, SMA strip actuator, Large deformation, Coupled Thermomechanical Analysis.

## 1. INTRODUCTION

*Shape memory alloys (SMA)* are one class of smart materials, which has the ability to show large deformation when loaded and recover their shape with the increase in temperature from the deformed shape, known as *Shape Memory Effect (SME)*. In *Pseudoelasticity (PE)* or *Superelasticity (SE)*, it undergoes a large deformation with increment in load and subsequently recover the same upon unloading. These features are harnessed in a wide variety of applications in the field of aerospace, robotics, biomedical, etc [1].

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The necessity of understanding and modelling of the thermomechanical response of SMAs led to the proposal of several constitutive models in the past four decades. In one class of approaches, e.g., Brinson [2], Tanaka [3], Liang and Rogers [4], the phase evolution is empirically derived from experiments, thus are often used for real time applications because of their simplicity. However, these possess limited functionality in the analyses of SMA based structures under practical thermomechanical loading cases. Whereas, in Lagoudas [1], the constitutive law as well as the phase evolution are derived based on the thermodynamic principles. It has been found that for the analyses of practical problems in 2D and 3D, under simultaneous variation of thermomechanical loading, the latter approaches are more suitable.

Vast majority of reported constitutive model considers the phase evolution, derived based on thermodynamic principles, taking temperature as an input variable. However, in reality, temperature is a state variable, that depends on applied thermomechanical loads, boundary conditions, and internal material properties. Besides, the endothermic and exothermic nature of the phase transformation affects thermal response of the SMA and thus renders thermomechanical coupling essential. In addition, SMA based components also undergo large deformation during thermomechanical loading. So, to model the response of the SMA based structures effectively, a fully coupled thermomechanical finite element based analysis tool, considering large deformation, is needed. To the best of authors' knowledge, limited number of studies have been reported, on the same, by researchers [5] using commercial finite element software, ABAQUS.

Alipour *et al.* [6] reported the thermo-mechanical response of an SMA wire under different loading conditions, using one-dimensional constitutive model of Brinson [2]. However, the effect of material level coupling during phase transformation was not considered in this formulation. Thiebaud *et al.* [7] implemented the SMA constitutive model of Raniecki and Lexcellent [8] by incorporating the heat equation in the COMSOL Multiphysics FE package. Under the isothermal condition, the SMA response of a plate in biaxial-tension and in-plane bending is simulated and validated using experimental data. Yang and Xu [9] incorporated the constitutive model of Seelecke and Muller [10] along with the heat conservation equation to simulate an SMA beam response under a tip force loading in COMSOL Multiphysics FE package. In 2014, Solomou *et al.* [5] proposed a 2D beam finite element, following Lagoudas *et al.* [11], for the coupled thermomechanical analysis of shape memory alloy actuators. First-order shear deformation theory (FSDT) was used, assuming the temperature distribution along the thickness direction as cubic and six-order [12] polynomial. Xu *et al.* [13] presented the effect of large deformation and rotation by considering logarithmic strain and Kirchhoff tress definition. The response of uniaxial bar and SMA torque tube is simulated using the developed formulation.

This article reports a fully coupled thermomechanical analysis of SMA beam actuator considering large deformation. The constitutive model of Qidwai and Lagoudas [14] is implemented in an incremental based non-linear finite element framework adopting updated Lagrangian (UL) formulation [15]. To consider the effect of large deformation, the Green-Lagrange strain-displacement relationships and 2$^{nd}$ Piola-Kirchhoff stress measures are considered. Both the mechanical and thermal equilibrium equations are solved simultaneously considering material level coupling terms using Newton-Raphson (NR) iterative scheme. Introduction of heat equation with coupling terms fetches the much needed time scale in the transient response of the system. The proposed formulation to predict the response under practical thermomechanical loading, is implemented in an open source finite element platform, deal.II [16].

The organization of the current paper is as follows. First, the constitutive modeling of SMA is discussed briefly, followed by a coupled finite element formulation. Section 3 illustrates the implementation of the mentioned formulation in the case of an elastic beam actuated by a SMA strip actuator. Next, the uncoupled FE model has been validated following the simulation results available in the literature. Finally, the coupled thermomechanical response of a SMA strip actuator is simulated to establish the effect of coupling terms and large deformation in the case of phase transformation.

## 2. MODELLING OF SHAPE MEMORY ALLOY

The constitutive model of Qidwai and Lagoudas [14] consists of 2 parts, (i) Constitutive relation (equation (1)), relating Second Piola-Kirchhoff stress ($S_{ij}$) with Green-Lagrange strain ($\varepsilon_{ij}$), and temperature ($T$) using internal material parameters, e.g., martensitic volume fraction ($\xi$) and transformation strain ($\varepsilon^t_{ij}$), respectively, (ii) Flow rule (equation (2)) depicting evolution of $\varepsilon^t_{ij}$ with $\xi$ subjected to a constraint (equation (3)), expressed in tensorial notation as,

$$\textit{Constitutive Relation}: \quad \mathrm{S}_{ij} = \mathbb{S}^{-1}_{ijkl}\left[\boldsymbol{\varepsilon}_{kl} - \alpha_{kl}(T - T_0) - \boldsymbol{\varepsilon}^t_{kl}\right], \tag{1}$$

$$\textit{Evolution Equation}: \quad \dot{\boldsymbol{\varepsilon}}^t_{ij} = \Lambda_{ij}\,\dot{\xi}, \tag{2}$$

$$\textit{Constraint Equation}: \quad (\mathrm{S}_{ij} : \Lambda_{ij} - \rho\frac{\partial G}{\partial \xi})\,\dot{\xi} = \pi\,\dot{\xi} \geq 0. \tag{3}$$

Here, $\Lambda_{ij}$ denotes the transformation direction tensor, determining the evolution of transformation strain considering the flow direction (forward or backward) and is assumed to have the following form,

$$\Lambda_{ij} = \begin{cases} \sqrt{\frac{3}{2}}H\frac{\mathrm{S}'_{ij}}{||\mathrm{S}'_{ij}||} & ; \quad \dot{\xi} > 0; \\ H\frac{\varepsilon^{t-r}_{ij}}{\bar{\varepsilon}^{t-r}} & ; \quad \dot{\xi} < 0\ . \end{cases} \tag{4}$$

Here, H denotes the maximum uniaxial transformation strain and $\varepsilon^{t-r}_{ij}$ is the transformation strain at the reversal of phase transformation. $S'_{ij}$ , $\bar{\varepsilon}^{t-r}$ depict the deviatoric stress tensor and the second invariant of transformation strain at reversal point, respectively.

## 3 FINITE ELEMENT IMPLEMENTATION

To take into account the large displacement and large deformation, Green-Lagrange strain measure is considered, which is related to displacement as,

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{i,k}u_{k,j}) = e_{ij} + \eta_{ij}, \quad (5)$$

where, $e_{ij}$ = 1/2 ($u_{i,j}$ + $u_{j,i}$), and $\eta_{ij}$ = 1/2 ($u_{i,k}u_{k,j}$) . In updated Lagrangian framework, the virtual work principle can be written as [17],

$$\int_{V_n} {}^{n+1}_{\;\;n}\mathbf{S}_{ij}\, \delta\, {}^{n+1}_{\;\;n}\varepsilon_{ij}\, dV = \underbrace{\int_{V_n} f^{n+1}_{b_i}.\, \delta u_i\, dV}_{\text{Body force}} + \underbrace{\int_{\Gamma_n} f^{n+1}_{t_i}.\, \delta u_i\, d\Gamma}_{\text{Traction force}}, \quad (6)$$

where, ${}^{n+1}{}_{n}()$ represent the quantity with in the parenthesis at $t_{n+1}$ calculated in the configuration at time $t_n$. $S_{ij}$ represents second Piola-Kirchhoff stress and $\delta_{ui}$ is the virtual displacement. ${}^{n+1}_{\;\;n}\varepsilon_{ij} = {}_{n}\Delta\varepsilon_{ij}$, is the increment in strain at configuration $t_n$ . Using incremental stress decomposition,

$${}^{n+1}_{\;\;n}\mathbf{S}_{ij} = {}^{n}_{n}\mathbf{S}_{ij} + {}_{n}\Delta\mathbf{S}_{ij} = {}_{n}\sigma_{ij} + {}_{n}\Delta\mathbf{S}_{ij} \;, \quad (7)$$

where, ${}_{n}\Delta S_{ij}$ is the increment in second PK stress, obtained from the increment in strain ( ${}_{n}\Delta\varepsilon_{ij}$ ) and temperature (${}_{n}\Delta T$ ) using continuum moduli tensors,

$${}_{n}\Delta\mathbf{S}_{ij} = {}_{n}\mathbb{L}_{ijkl} : {}_{n}\Delta\varepsilon_{kl} + {}_{n}\mathcal{T}_{ij}\, {}_{n}\Delta T. \quad (8)$$

Where, ${}_{n}()$ is the calculated parameter in the parenthesis at time $t_n$. In presence of the coupling terms, the governing heat balance equation is expressed as,

$$\rho C\dot{T} = KT_{,ii} + Q_s + Q_L + Q_T \;\;, \quad (9)$$

where, $Q_L$ and $Q_T$ refer as the heat energy release or absorption per unit volume due to latent heat at phase transformation and thermoelastic effect, respectively. $Q_s$ depicts the heat energy input per unit volume and $K$ signifies the conductivity of the domain. $\rho$ and $C$ correspond to density and specific heat, respectively. Following Lagoudas *et al.* [11], these terms can be expressed as,

$$Q_L(\mathrm{S}_{ij}, T, \xi) = (\pi - \frac{\partial \pi}{\partial T}T)\dot{\xi}\;;\;\; Q_T(\mathrm{S}_{ij}, T, \xi) = -\alpha_{ij} : \dot{\mathrm{S}}_{ij}T\;. \quad (10)$$

Using time discretization procedure following Crank-Nicolson method [18], equation (9) can be discretised with respect to time as,

$$\rho C\left(\frac{T^{n+1} - T^{n}}{\Delta t_n}\right) = \theta\psi^{n+1} + (1-\theta)\psi^{n}, \quad (11)$$

where, $\psi = [K_{T,ii} + Q_s + Q_L + Q_T]$. The superscript $n$ indicates function evaluations at time $t_n$ (similarly for $n + 1$) and $\Delta t_n = t_{n+1} - t_n$, is the time step size, $\theta$ has been chosen as 0.5. $Q_L$ and $Q_T$ (equation (9)) are nonlinear terms depending on stress, temperature. After linearization with respect to stress and temperature at $n^{th}$ step one gets,

$$(Q_L + Q_T)^{n+1} = (Q_L + Q_T)^n + \mathcal{X}_{ij}^n \, {}_n\Delta \mathrm{S}_{ij} + \mathcal{Y}^n \, {}_n\Delta T \; . \tag{12}$$

where, ${}_n\Delta \mathrm{S}_{ij}$ and ${}_n\Delta T$ are the increment in second PK stress and temperature, respectively at $n^{th}$ step and

$$\mathcal{X}_{ij}^n = \frac{\partial}{\partial S_{ij}}(Q_L + Q_T)^n; \;\; \mathcal{Y}^n = \frac{\partial}{\partial T}(Q_L + Q_T)^n.$$

By writing, ${}_n\Delta\varepsilon$ and ${}_n\Delta T$ in terms of nodal increment in displacement ($\Delta$U) and nodal increment in temperature ($\Delta T$) degrees of freedom, respectively, the discrete form of the thermal equilibrium equation appears to be,

$$\boldsymbol{K}_{TU}\Delta\boldsymbol{U} + \boldsymbol{K}_{TT}\Delta\boldsymbol{T} = \Delta\boldsymbol{F}_T, \tag{13}$$

where, $\boldsymbol{K}_{\mathrm{TU}}$ and $\boldsymbol{K}_{\mathrm{TT}}$ relate residual thermal load vector ($\Delta\boldsymbol{F}_{\mathrm{T}}$) with increment in nodal displacement and temperature, respectively. Similarly, the discrete form of the stress equilibrium equation yields,

$$\boldsymbol{K}_{UU}\Delta\boldsymbol{U} + \boldsymbol{K}_{UT}\Delta\boldsymbol{T} = \Delta\boldsymbol{F}_U. \tag{14}$$

Here, $\boldsymbol{K}_{\mathrm{UU}}$ and $\boldsymbol{K}_{\mathrm{UT}}$ relates residual mechanical load vector ($\Delta\boldsymbol{F}_{\mathrm{U}}$) with increment in nodal displacement and temperature, respectively. Finally, combining equation (13) and (14), the system of equations can be represented as,

$$\underbrace{\begin{bmatrix} \boldsymbol{K}_{UU} & \boldsymbol{K}_{UT} \\ \boldsymbol{K}_{TU} & \boldsymbol{K}_{TT} \end{bmatrix}_n}_{\text{Tangent Stiffness Matrix}} \underbrace{\begin{Bmatrix} \Delta\boldsymbol{U} \\ \Delta\boldsymbol{T} \end{Bmatrix}_{n+1}}_{\text{Incremental DOFS}} = \underbrace{\begin{Bmatrix} \Delta\boldsymbol{F}_U \\ \Delta\boldsymbol{F}_T \end{Bmatrix}_{n+1}}_{\text{Residue}} . \tag{15}$$

Equation (15) has to be solved iteratively at each time step following Newton-Raphson method (Figure 1), till the residual of mechanical and thermal load vectors are driven to zero.

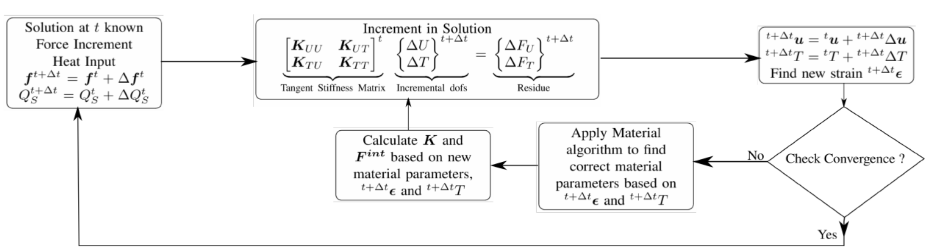


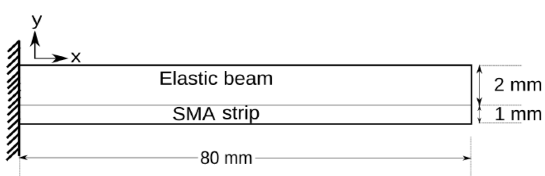

Figure 1: Flowchart for coupled thermomechanical analysis

## 4. SIMULATION OF SMA ACTUATED SHAPE ADAPTIVE STRUCTURE

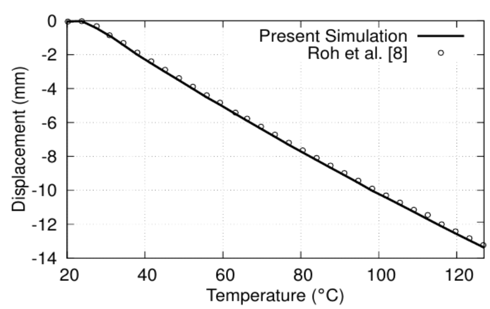
To highlight the capability of the formulation, the interaction between a SMA strip actuator and an elastic host structure, made up of aluminium (E = 69 GPa, ν =0.33), is investigated. The SMA strip is used to achieve bending actuation of the host beam. The material parameters of SMA is listed in Table 1. Initially, a transformation strain of 5% is imparted to the SMA strip uniformly, in x direction, and is fixed to the elastic host beam along its length, as shown in Figure 2(a). Firstly, to verify the developed FE model, the thermomechanical response of the structure is simulated by increasing the input temperature to 125 ˚C, following Roh *et al.* [19]. The mesh and time convergence study is performed to find the suitable mesh density and time step to be used in the following analysis, respectively. Following this, the domain is discretised using 50 (along length) × 10 (along height), and a time step of 1/20 sec is considered for the analysis. The simulated response is found to be in complete agreement with the same reported in the literature [19], as depicted in Figure 2(b).

| | | | |
|---|---|---|---|
| $A_s = 295$ K | $A_f = 315$ K | $M_s = 291$ K | $M_f = 271$ K |
| $k = 18$ WK$^{-1}$m$^{-1}$ | $E^A$=70.0 GPa | $E^M$=30.0 GPa | $c^A = c^M = 470$ Jkg$^{-1}$K$^{-1}$ |
| $C^A = C^M = 70$ MPaK$^{-1}$ | $\rho = 6500$ kgm$^{-3}$ | $\nu = 0.3$ | $H = 0.05$ |

Table 1: Shape Memory alloy material parameters [19]



(a)



(b)

Figure 2: (a) Aluminium beam structure actuated by a SMA strip (Roh *et al.* [19]), (b) displacement response with applied temperature.

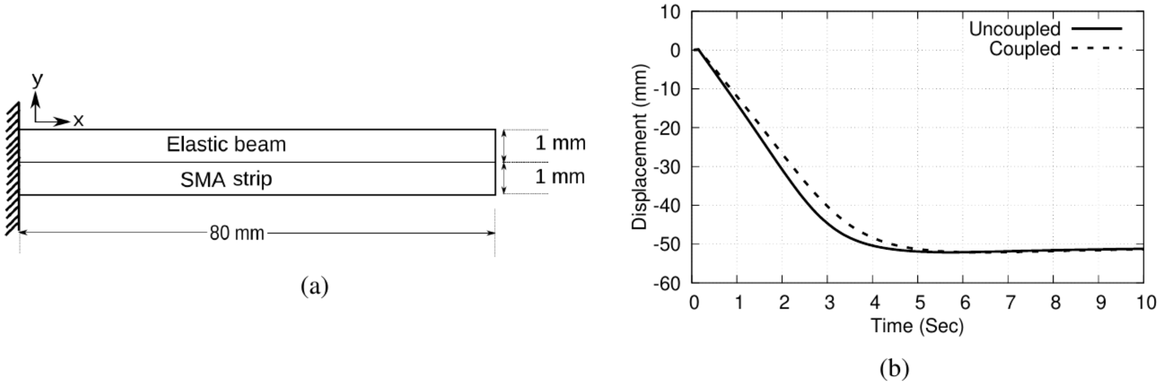



Figure 3: (a) Aluminium beam to SMA strip thickness ratio of 1:1, (b) comparison between displacement responses between coupled and uncoupled models.

Then, to investigate the effect of material level coupling while undergoing large deformation, SMA strip to elastic host's thickness ratio of 1:1 is considered (shown in Figure 3(a)) and an uniform heat source of 50 W/mm$^2$ is applied to activate the SMA strip, causing bending of the structure. As shown in Figure 3(b), a significant difference in displacement response is observed between the coupled and uncoupled formulation, due to the endothermic nature of reverse transformation. In addition, the transient response of the system is found to depend on the heating rate and thermomechanical loading path. Figure 4 illustrates the corresponding beam configurations at different time instants, elucidating the discrepancy in the responses obtained from the coupled and uncoupled models.

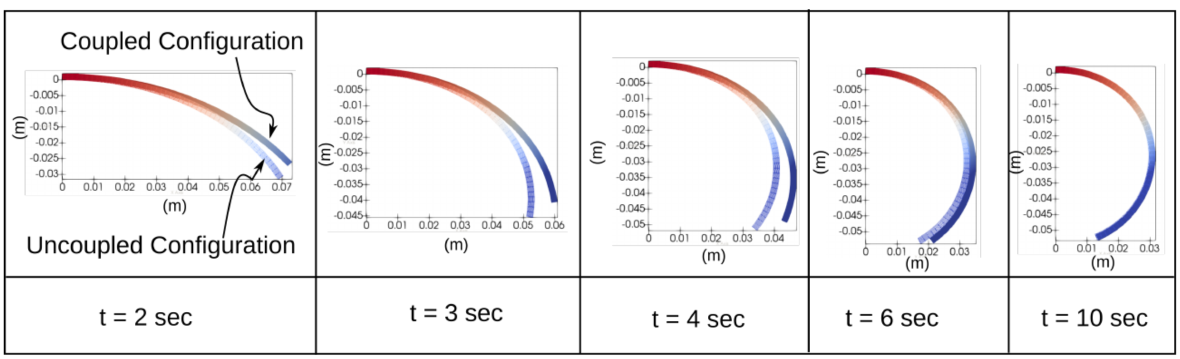



Figure 4: Configuration of the structure at different time instants.

## 5 SUMMARY AND CONCLUSION

In this work, a thermomechanically coupled formulation for SMA based structures is developed and implemented in a non-linear finite element framework. The effect of the non-linear terms, i.e., latent heat of phase transformation and thermoelastic heat due to stress rate are explored in this formulation considering large deformation. Consideration of mechanical loading, applied heat source and boundary conditions significantly alter the performance of the system. To effectively predict the response of SMA based components, the effect of latent heat is essential to capture the effect of exothermic or endothermic nature of transformation, especially during phase transformation. In addition, the effect of large deformation is also investigated by considering proper stress and strain measures. This formulation can be extended for other 2D and 3D applications, such as SMA stents, origami structures, which will be explored in future.

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