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Numerical approximation of buoyancy driven turbulent heat and mass transfer past a vertical plate using LRN k-ε model

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Abstract: The authors studied the transient natural convective heat and mass transport of a turbulent flow adjacent to the vertical plate using low Reynolds number(LRN) k-ε model. The research problem on turbulent flow is assumed to be two dimensional and viscous incompressible. The conservation equations such as average continuity, momentum, energy and concentration along with additional transport equations turbulent kinetic energy (TKE) and dissipation rate of TKE are considered as per the flow geometry. Due to the non-availability of analytical or direct numerical techniques, the corresponding highly nonlinear PDE's are solved by adopting the standard implicit type of finite difference method namely Crank-Nicolson scheme. Because, this scheme is unconditionally stable and helps to keep the governing turbulent flow equations in discretized form and are solved via tridiagonal matrix algorithm. The simulated results studied with different control parameters, i.e. Re_t, Pr_t, Sc_t and Bu_t are graphically displayed and also analyzed the time averaged velocity, temperature, concentration, turbulence energy and dissipation rate of TKE profiles. To understand the physical phenomenon of turbulent flow, the authors also shown the turbulent behaviour of average momentum, energy and mass transfer rates. It has been noted that, the average temperature and concentration fields boosted with rising values of Re_t. Also, TKE and dissipation rate of TKE profiles increase with enhancing turbulent Bu_t values. In addition, the simulated turbulent flow results from the LRN k-ε model are compared with usual laminar flows as a special case and found to be in respectable agreement.

Keywords: Turbulent energy and mass transfer, Vertical plate, k-ε model, Low Reynolds number, Free convection.

1. Introduction: In the field of engineering applications, turbulence models have been researched enormously due to its vital role in energy and mass transport, dispersion and mixing, surface drag, and momentum transport, etc. Particularly, turbulent flows occur in the process of determining the heat and mass transport in chemical engineering problems, affecting the chemical reactions and its performance. The proposed transient buoyancy driven turbulent energy and mass transfer flow around the vertical plate have an excellent application in the industry and manufacturing process such as hot filaments, heat exchangers, and nuclear reactors, etc. The effects of transient laminar energy and mass diffusion across a vertical plate/cylinder have been studied by many researchers [1-4]. Most of the research work done on the laminar boundary layer flows which can be continued to the turbulence flows since the available literature related to boundary layer turbulent flows is very less. In this scenario, Fedorov and Viskanta [5] used LRN k-ε method to investigate the transient buoyancy driven turbulent energy transport across an asymmetrically heated parallel plate channel. Using the same model, Fedorov et al. [6] investigated the turbulent mass transfer for the vertical parallel plate channel geometry. Similarly, many other researchers have done theoretical investigations on turbulent heat transfer with different turbulence models have been proposed by Anilkumar et al. [7], Claudia and Rodion [8], Tao Zhi et al. [9], Rincon-Casado et al. [10], etc. Recently, the k-ε turbulent flow model was used by Ahmed and Rachid [11] to analyze the turbulence and energy transfer in tube with heat exchanger & shell using finite volume numerical method. Also, for the investigation of turbulent liquid flow and boundary layer turbulent energy diffusion in pseudoplastic fluids over a plate, Chanjuan et al. [12] applied the similarity transformations.

The authors concluded from the literature review that, the research on unsteady natural convective turbulent heat and mass transport has received very little attention. As a result, influenced by previous research on turbulent flow and work recommended by Fedorov et al. [6], the authors in the current paper used the LRN turbulent k-ε model to simulate turbulent energy and mass transport flow from a vertical plate.

2. Mathematical model: The considered physical problem is stationary and it studies the time dependent free convective turbulent heat and mass transfer flow over a semi-infinite vertical hot plate as displayed in Fig. 1. A rectangular region of geometry is assumed to characterize the turbulent flow problem in the Cartesian coordinate system, where the x-axis is measured in the axial direction and the y-coordinate is measured normal to the vertical plate. The free stream average thermal ($\bar{T}_\infty$) and concentration ($\bar{C}_\infty$) values are equivalent to the wall temperature ($\bar{T}_w$) and wall concentration ($\bar{C}_w$) fields at a time $t' = 0$. As time approaches to $t' > 0$, the average thermal and concentration fields grew to $\bar{T}_w$ ($> \bar{T}_\infty$) and $\bar{C}_w$ ($> \bar{C}_\infty$) respectively and maintained constant for all the time $t' > 0$. It is worth noting that natural boundary layers aren't just found in laminar flows; they may also be found in turbulent flows. As a result of the average temperature and concentration differences at the plate surface, the density variations occur and interact with the gravity field to create free convective turbulent heat-mass transfer flow over the vertical plate. Using Boussinesq's approximation, the resulting set of two-dimensional nonlinear time averaged Navier-Stokes equations for the conservation of mass, momentum, energy and concentration are specified according to turbulent fluid flow and geometry. With these flow configurations and assumptions, the Reynolds averaged Navier-Stokes governing equations including equation of kinetic energy and its dissipation rate of the turbulent flow can be shown as follows [6].

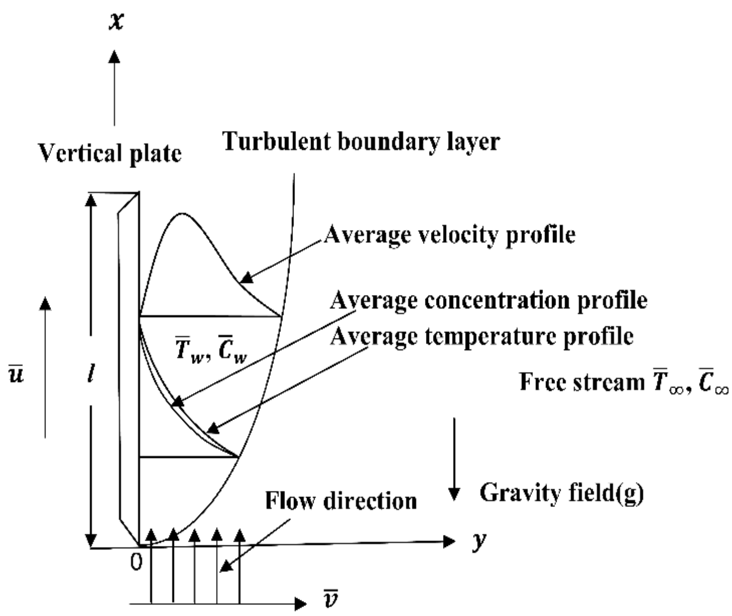

Figure 1: Geometry and coordinate system of the investigated problem.

Conservation of mass:

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \tag{1}$$

Conservation of momentum:

$$\frac{\partial \bar{u}}{\partial t'} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{v}}{\partial y} = \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial \bar{u}}{\partial y}\right] + g\beta_T(\bar{T} - \bar{T}_\infty) + g\beta_C(\bar{C} - \bar{C}_\infty) \tag{2}$$

Conservation of energy:

$$\frac{\partial \bar{T}}{\partial t'} + \bar{u}\frac{\partial \bar{T}}{\partial x} + \bar{v}\frac{\partial \bar{T}}{\partial y} = \frac{\partial}{\partial y}\left[\left(\alpha + \frac{\nu_t}{\sigma_T}\right)\frac{\partial \bar{T}}{\partial y}\right] \tag{3}$$

Conservation of species:

$$\frac{\partial \bar{C}}{\partial t'} + \bar{u}\frac{\partial \bar{C}}{\partial x} + \bar{v}\frac{\partial \bar{C}}{\partial y} = \frac{1}{Sc_t}\frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial \bar{C}}{\partial y}\right] \tag{4}$$

In Eqs. (1)-(4), ν_t is the turbulent kinematic viscosity, which is calculated using the turbulent kinetic energy (k) and turbulent energy dissipation rate (ϵ). Furthermore, using the LRN k-ε model, the transport equations for k and ϵ are derived from the Navier-Stokes equations of motion using Reynolds decomposition [13, 14] and are provided as follows:

Equation of turbulence energy:

$$\frac{\partial k}{\partial t'} + u\frac{\partial k}{\partial x} + v\frac{\partial k}{\partial y} = \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right] + \nu_t\left(\frac{\partial \bar{u}}{\partial y}\right)^2 - \epsilon - 2\nu\left(\frac{\partial k^{1/2}}{\partial y}\right)^2 \tag{5}$$

Equation of dissipation rate of turbulence energy:

$$\frac{\partial \epsilon}{\partial t'} + u\frac{\partial \epsilon}{\partial x} + v\frac{\partial \epsilon}{\partial y} = \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_\epsilon}\right)\frac{\partial \epsilon}{\partial y}\right] + C_1\frac{\epsilon \nu_t f_1}{k}\left(\frac{\partial \bar{u}}{\partial y}\right)^2 - C_2 f_2\frac{\epsilon^2}{k} + 2\nu\nu_t\left(\frac{\partial^2 \bar{u}}{\partial y^2}\right)^2 \tag{6}$$

The turbulent eddy viscosity (ν_t) is related to k and ϵ in the above conservation equations and k-ε turbulence model is given by

$$\nu_t = C_\mu \frac{k^2}{\epsilon} f_\mu \tag{7}$$

Also, 0.09, 1.45, 2.0, 0.9, 1.0 and 1.3 are the appropriate constant values for C_μ, C_1, C_2, σ_T, σ_k and σ_ϵ, respectively. Further, the low Reynolds number wall damping functions $f_1 = 1$, $f_2 = 1 - 0.3\exp\left(-{Re_t}^2\right)$ and $f_\mu = \exp\left(\frac{-2.5}{1+\frac{Re_t}{50}}\right)$ including turbulent Prandtl (Pr_t) and Schmidt (Sc_t) numbers are not well suited for turbulent natural convective flows, although Jones and Launder [15], Patel et al. [16] have been produced and adjusted for forced convection boundary layer flows. The values of forced convection flows are employed in the current study for the natural convection due to a lack of better options.

For the specified rectangular geometry, the initial and boundary conditions of turbulent heat and mass transfer flow are as follows [17]:

$t' \leq 0$; $\quad \bar{u} = 0,\ \bar{v} = 0,\ k = 0,\ \epsilon = 0,\ \bar{T} = \bar{T}_\infty,\ \bar{C} = \bar{C}_\infty$ for all x and y

$$t' > 0\,; \qquad \bar{u} = 0,\ \bar{v} = 0,\ k = 0,\ \epsilon = 0,\ \bar{T} = \bar{T}_w,\ \bar{C} = \bar{C}_w \ \text{ at } y = 0 \tag{8}$$

$$\bar{u} = 0,\ \bar{v} = 0\,,\ k = 0,\ \epsilon = 0,\ \bar{T} = \bar{T}_\infty,\ \bar{C} = \bar{C}_\infty \ \text{ at } \ x = 0$$

$$\bar{u} \to 0,\ \bar{v} \to 0,\ \ k \to 0,\ \epsilon \to 0,\ \bar{T} \to \bar{T}_\infty, \bar{C} \to \bar{C}_\infty \text{ as } y \to \infty$$

3. Numerical approach: For the current problem, time averaged governing coupled equations are illustrated and numerical results are obtained by adopting the Crank-Nicolson scheme of implicit finite difference method. The computed turbulent flow data displayed graphically and examined in terms of time average velocity, temperature, concentration, kinetic energy and dissipation rate in addition to turbulent average momentum, heat & mass transport rates using various turbulent control parameters.

4. Conclusions:

The FDM results have been carried out for the unsteady natural convective turbulent heat and mass transfer flow along the heated semi-infinite vertical plate using a low Reynolds number k-ε turbulence model. The coupled turbulent governing equations are derived and solved numerically using the implicit Crank-Nicolson scheme of the finite difference method. The computations are carried out for different turbulent controlled parameters such as Re_t, Sc_t and Bu_t. The simulated results are displayed graphically and analyzed in terms of average turbulent velocity, temperature, concentration, kinetic energy, dissipation rate and friction parameters.

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