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Detection of edge crack in beam like structure modeled as rotational spring by using Bayesian filtering

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ABSTRACT

Numerical beam is an important structure type for replicating and idealizing several different real structures, such as bridges, high-rise structures etc. Towards development of damage detection approaches/ algorithms for bridge structure, present study has idealized it as a prismatic beam and an instance of localized structural damage is replicated as a through crack in the beam. The objective of this study is further to precisely localize the extent and location of such damages under severe ambient and model uncertainties. Particle filtering based structural health monitoring (SHM) approach has been employed in this study. Typical of such model-based algorithms, the real structural domain is discretized into finite elements with the damage attributed to any of these discrete elements, rendering the localization precision being dependent on the discretization resolution. Current study modelled the damage as an independent massless rotational spring in the beam attributing the local stiffness deterioration of the real structure through the decay in the spring stiffness. To localize and quantify the real damage in the beam, the goal of present estimation approach is therefore set as to locate the position of such spring and further estimate its residual spring stiffness. This enables localization of the damage with a resolution much beyond the discretization density, and thereby helping the algorithm to be supported with much less compute-intensive models. The algorithm is validated for its sensitivity against noise and damage severity. The results demonstrate the algorithm to be efficient, prompt and precise in detecting damage in a structure.

Keywords: Particle filtering, Structural health monitoring, mass-less rotational spring ,finite element model, model-based algorithm

1. INTRODUCTION

Real life infrastructures undergo deterioration in their stiffness due to ageing or sudden damage induced through a catastrophic loading condition, like seismic excitation, tsunami, blast or mere unfortunate accidents. While the ageing related deterioration is slow processed and therefore may give the user ample time to realize the same, the sudden damages may cause a catastrophic structural failure without raising an alarm. The time window between the first damage occurrence and its consequent progression to complete failure may at times do not allow sufficient time to the user or owner of the infrastructure to undertake any rigorous health assessment procedure. Eventually, this necessitates for a real time approach to detect and locate the damage in real time.

A crack/damage in an operational civil infrastructure is ideally to be detected and isolated as soon as it occurs in order to avoid further damage progression, economic loss and loss of precious human lives. With model assisted structural health monitoring (SHM) approach, the detection resolution, however, depends on spatial discretization of the overall structural domain achieved using numerical models like finite element model (FEM) and also on the density of the sensors with which the structural response has been sampled [Chasalevris and Papadopoulos(2006), Ghadami et al.(2013)Ghadami, Maghsoodi, and Mirdamadi]. Accordingly, for high dimensional civil infrastructure, employment of high dimensional supporting model and dense instrumentation becomes imperative which might overshoot the cost of monitoring rendering the entire approach economically not viable. Eventually, there is a practical need to address these issues with the real life SHM to make it applicable for the existing high dimensional infrastructures. Driven with an objective to develop a practical SHM approach, present study aims for an innovative approach wherein high dimensional structures can still be estimated using cheaper computational models and sparse instrumentation.

2. LITERATURE REVIEW

For the model-based damage detection approaches, damage/crack in a structure has generally been replicated in a numerical model through parameterizing the stiffness of one or more discretized elements with the crack features, like depth, location, shape etc. Eventually, such detection problems are a two step approach: modelling the crack intended to be detected through a model, (sometimes validated in frequency domain against real experimental results) and employing inverse approach to estimate the crack parameters (in time or frequency domains [Lee(2009)]) taking basis on the support model in an iterative process [Lee(2009)]. In frequency domain, the frequency contour method has been a favourable tool for identification of single crack using first three natural frequencies [Patil and Maiti(2003)]. This approach was further extended by [Liang et al.(1991)Liang, Choy, and Hu] to deal with multiple crack scenarios. [Liang et al.(1991)Liang, Choy, and Hu] modelled the crack as a rotational mass-less spring of infinitesimal length. [Hu and Liang(1993)] in a similar approach replicated a crack as a mass-less spring in a discrete model of a beam. [Aswal et al.(2022)Aswal, Sen, and Mevel] employed end fixity factor, which in turn is a function of stiffness reduction factor γ, to model damage in a fixed joint.

It has been perceived, that research in this field has typically been undertaken in frequency domain to estimate crack depth and location. The underlying presumption has been that global dynamic properties like modal frequencies and mode shapes can be severely affected by the reduced flexural rigidity (EI) of a crack section. This assumption is however very crude since for most SHM problems the cracks capable of affecting global properties are usually visible with bare eyes. Eventually, the kinds of problems for which SHM is necessary deal with fine cracks that can only affect the dynamics locally. Frequency domain approaches are therefore less practical for such cases and application of time domain approaches becomes imperative.

Nevertheless, time domain approach with real time detection capabilities are severely restricted by the real time computation demand and required instrumentation. High dimensional structures monitored with real time algorithm demands an exorbitant amount of real time computation which in turn depends on the discretization resolution of the support models. Eventually, the model dimensionality should be managed within acceptable limits: not too coarse to loose damage localization resolution, not too fine to end up with uneconomical computation demand. This study, handles this problem in a hierarchical process. In this approach, the real structure is replicated with a coarsely discretized model that will be used to close down to the damage location. A further modelling within the located damaged element will parameterize the exact damage location within the element and depth.

The proposed approach has been verified within a context of a beam which is a very interesting structure since some major civil engineering structures can actually be replicated with such simple representation, provided the stiffness and mass distribution has been properly calibrated. For example, a high-rise building can satisfactorily replicated with a cantilever beam of non uniform stiffness (and/or mass distribution), via calibration using real structural response. Similarly, a bridge can be idealized as a simply supported or continuous beam depending on the actual case. This article validates the proposal on a simplistic beam structure while further investigation involving real infrastructures will be presented in the future communications.

Finally, to address the real life uncertainties originating from the model inaccuracies and sensor noises, the proposal has been defined in a probabilistic domain. Bayesian filtering is an efficient recursive framework for stochastic system estimation and has been employed in this article for identification. By structure, damage detection problems are typical joint state-parameter estimation problems [Kuncham et al.(2022)Kuncham, Sen, Kumar, and Pathak] and of nonlinear in nature regardless of the linearity condition of the actual structure [Sen et al.(2018)Sen, Crinière, Mevel, Cérou, and Dumoulin]. In such estimation problems, nonlinear and approximate filters are employed, such as Extended, Unscented, Ensemble Kalman filters and Particle filters. Eventually, Particle filter, a nonlinear Bayesian filter variant is employed for estimating the crack through its parameters, i.e., location of cracked element, depth and position with respect to the cracked element.

However, PF is a compute-intensive crude approach wherein the uncertainty in the parameter space is propagated through a sample based approach [Chatzi and Smyth(2009)]. Yet, the dynamics of the actual structure under consideration is linear for which such nonlinear variant of Bayesian filter is not necessary and also economically not justified. Eventually, one can decouple the estimation of states (of linear dynamics) and parameters (crack features) and employ separate but interacting filtering approaches to achieve precision and economy. This approach is typically termed as interacting (or marginalized) filtering approach. In this study, the interacting Particle-Kalman filter (IPKF) approach introduced by [Zghal et al.(2014)Zghal, Mevel, and Del Moral, Sen et al.(2018)Sen, Crinière, Mevel, Cérou, and Dumoulin, Aswal et al.(2021)Aswal, Kuncham, Sen, and Mevel] has been employed to estimate the states and parameters simultaneously without sacrificing the economy.

3. METHODOLOGY

Typical model-based SHM approaches defines the structural domain in terms of finite elements and damage is attributed in one of the elements. Eventually, finer is the discretization, the precise is the detection resolution, especially for localized damages, like cracks. cracks have been modelled in this study as a mass-less rotational spring and its features like position and severity has been estimated using IPKF technique. In the next section, the crack modelling is detailed followed by a brief introduction of the employed IPKF approach.

3.1. Crack modelling

As discussed, cracks in this study are replicated as mass-less rotational springs of infinitesimal lengths [Ghadami et al.(2013)Ghadami, Maghsoodi, and Mirdamadi, Lee(2009)] reducing the flexural rigidity right where they are placed. The modelling of the damaged structure will be detailed in this study within a context of a beam of length (L) discretized into 'n' two-nodded Euler Bernoulli beam (EBB) elements with the damage located in one of the elements. The damage element is further modelled as two EBB joined through a mass-less rotational spring right where the crack is intended to be modelled. The crack induced reduction in rigidity (EI) and rotational stiffness (k_i) in a beam segment of length l can further be replicated as reduction in the joint stiffness of the spring following [Aswal et al.(2022)Aswal, Sen, and Mevel] in terms of stiffness reduction factor (γ)and reported as Equation 1,

$$k_i = \gamma \frac{EI}{l} \tag{1}$$

with γ varying within the range of $[0.5 - 25]$ (Pinned: $\gamma < 0.5$, flexible joint: $0.5 < \gamma < 8$, rigid joint: $\gamma >= 25$). End fixity factor 'F_i' is further correlated to γ as $F_i = \gamma/(\gamma + 3)$ [Aswal et al.(2022)Aswal, Sen, and Mevel].

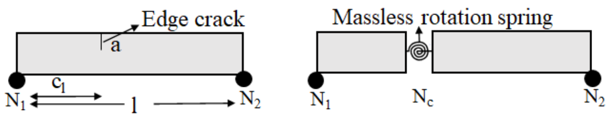


Figure 1. Schematic diagram of modelled crack element of the beam

With such model for crack (c.f Figure 1), the finite element approximation however changes with a modified mass and stiffness matrix (M_{cr} and K_{cr}) with their dependence on the end-fixity factor(F_i) (cf. [Aswal et al.(2022)Aswal, Sen, and Mevel] for further details).

This article has assumed coarse discretization for the overall structure using two dimensional two nodded (four *dof*s) EBB elements. However, for the damaged element, the element is further defined as two of such EBB elements conjoined with a rotational spring in between. The stiffness of this two-element six *dof*s subdomain is further dynamically reduced down to four *dof*s through Structure Equivalent Reduction Expansion Program (SEREP). The stiffness of this subdomain is parameterized with the position of this rotational spring (denoting precise location of the crack) and stiffness reduction factor γ for the spring (denoting severity of the damage). Eventually, this allows perfect localization and quantification of the crack within the damaged element.

3.2. Interacting Kalman-particle filtering (IKPF)

Interacting filtering strategy is an efficient approach for stochastic system estimation wherein states and parameters are individually estimated conditioned on each other. IPKF approach introduced by [Zghal et al.(2014)Zghal, Mevel, and Del Moral] and later improved and validated for mechanical system by [Sen et al.(2018)Sen, Crinière, Mevel, Cérou, and Dumoulin, Aswal et al.(2021)Aswal, Kuncham, Sen, and Mevel] employs PF and KF for parameter and state estimation respectively. Here in this study, the damage features like damaged element, location of the crack with respect to the damaged element and damage severity (crack depth) are considered as parameters while the system dynamics is estimated through estimation of the pertinent states (detailed in the following). Both estimation draws inference from a set of collocated/non-collocated measurements that are contaminated with stationary white Gaussian noise (SWGN).

Bayesian filtering based system estimation approaches, however, demands the system to be defined in state space. The governing differential equation for the system dynamics, with $(n \times n)$ order stiffness $(\mathbf{K})$, mass $(\mathbf{M})$ and damping $(\mathbf{C})$ as pertinent system matrices and $\mathbf{P}$ as external forcing, can be represented as:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{P}(t) \tag{2}$$

Where $\ddot{\mathbf{q}}(t)$, $\dot{\mathbf{q}}(t)$, and $\mathbf{q}(t)$ are the acceleration, velocity and displacement vectors of order $n \times 1$. The simulated response is eventually measured relative to the fixed support using a set of acceleration sensors. This system dynamics and the measurement when defined through a set of unobserved states of order $(2n \times 1)$ in state space and observed through a set of n outputs, can be represented as,

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{v}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) + \mathbf{w}(t) \end{aligned} \tag{3}$$

here $\mathbf{x}(t) = \begin{bmatrix} \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) \end{bmatrix}, \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M^{-1}K} & -\mathbf{M^{-1}C} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M^{-1}} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} -\mathbf{M^{-1}K} & -\mathbf{M^{-1}C} \end{bmatrix},$ $\mathbf{D} = \begin{bmatrix} \mathbf{M^{-1}} \end{bmatrix}, \mathbf{u}(t) = \mathbf{P}(t)$. $\mathbf{v}(t)$ and $\mathbf{w}(t)$ are the process and measurement noise respectively assumed to be following SWGN processes of constant covariance Q and R respectively. Since in reality, the responses are discretely sampled, the discrete time description of Equation 3 can further be reproduced with their corresponding discrete time entities as,

$$\begin{aligned} \mathbf{x}_k &= \mathbf{A}_k x_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{v}_k \\ \mathbf{y}_k &= \mathbf{C} x_k + \mathbf{D}\mathbf{u}_k + \mathbf{w}_k \end{aligned} \tag{4}$$

It should be noted that in the discrete time definition, the system matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are defined with additional dependency on the time step k. This is to signify their time variance nature originating from the time variance in stiffness matrix $\mathbf{K}$. This dependency is however not explicitly presented in equations before Equation (4) for the sake of readability. Time variance in $\mathbf{K}$ can be parameterized with the crack features as mentioned previously.

Eventually, this system can be estimated with the IPKF approach wherein KF estimates linear $\mathbf{x}_k$ while PF is employed for costly parameter estimation. This also helps in managing the overall state dimension and thereby reduces the possibility of ill-posedness. IPKF nests a bank of KF within an envelop of a PF, in which each KF is conditioned on one realization drawn from a particle set [ξ^1, ξ^2, ξ^3, ..., ξ^N]. The time evolution of each particle is defined as a random perturbation around its current position with mean ξ^j_{k-1} with a stretch $\delta\xi_k = (1-\alpha)\bar{\xi}_{k-1}$ and a spread of $\sigma^{\xi}_{\mathbf{k}}$. Using α , turbulence can be reduced in particle estimation by re-centering the particles towards their mean $\bar{\xi_{k-1}}$ as follows,

$$\xi^{\mathbf{i}}{}_{\mathbf{k}} = \alpha\xi^{\mathbf{i}}{}_{\mathbf{k-1}} + \mathbf{N}(\delta\xi^{\mathbf{i}}{}_{\mathbf{k}}; \sigma^{\xi}_{\mathbf{k}}) \tag{5}$$

Subsequently, the evolved particles are weighted based on their likelihood against the current measurement. For this, the evolution of the states are conditioned on the particle estimates through the system matrices $\mathbf{A},\ \mathbf{B},\ \mathbf{C},$ and $\mathbf{D}$. The propagated states are then observed as $\tilde{\mathbf{y}}_k$ (cf. Equation 4) and the innovation (i.e. departure from the actual measurement $\mathbf{y}_k$) is obtained. Accordingly, the likelihood estimate for each of the particle can be obtained by processing the innovation through a Gaussian likelihood function. The derivation is only briefed in this article for the sake of brevity and the detail can be found in [Sen et al.(2018)Sen, Crinière, Mevel, Cérou, and Dumoulin]. A pseudo-code, presented in Algorithm 1, can however be referred for more insight of the algorithm.

Algorithm 1 Proposed parameter estimation algorithm for 2D beam

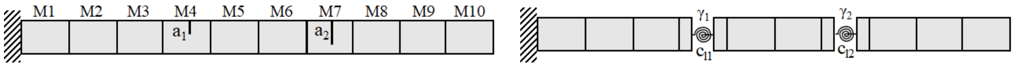
procedure IPKF($\mathbf{y}_k, \mathbf{Q}, \mathbf{R}$) ▷ Interacting particle and Kalman filter algorithm
  Initialize particles $\{\xi^j_0\}$, and state estimates $\{\mathbf{x}^{i,j}_{0|0}\}$ ▷ Initial guess
  for `<each` k^{th} `measurement` $\mathbf{y}_k$`>` **do**
    procedure IPKF($\{\xi^j_{k-1}\}, \{\mathbf{x}^{i,j}_{k-1|k-1}\}$) ▷ Initiating Kalman filter
      for `<each particle` ξ^j_k`>` **do** ▷ Initiating particle filter
        Evolve $\{\xi^j_{k-1}\} \rightarrow \{\xi^j_k\}$
        procedure KF($\xi^j_k, \{\mathbf{x}^{i,j}_{k-1|k-1}\}, \mathbf{y}_k$) ▷ For each j^{th} particle
          $\mathbf{x}^j_{k|k-1} = \mathbf{A}^j_k x^j_{k-1|k-1}$ ▷ Predicted state
          $\mathbf{P}^j_{k|k-1} = \mathbf{A}^j_k \mathbf{P}^j_{k-1|k-1} \mathbf{A}^{jT}_k + \mathbf{B}^j_k \mathbf{Q}_k \mathbf{B}^{jT}_k$ ▷ State covariance
          $\mathbf{K}^j_k = \frac{\mathbf{P}^j_{k|k-1}\mathbf{H}^T_k}{\mathbf{H}_k \mathbf{P}^j_{k|k-1}\mathbf{H}^T_k + \mathbf{R}_k}$ ▷ Kalman gain
          $\epsilon^j_{k|k-1} = \mathbf{y}_k - \mathbf{y}^j_{k|k-1}$ ▷ Innovation
          $\mathbf{x}^j_{k|k} = \mathbf{x}^j_{k|k-1} + \mathbf{K}^j_k \epsilon^j_{k|k-1}$ ▷ Updated predicted state
          $\mathbf{P}^j_{k|k-1} = (\mathbf{I} - \mathbf{K}^j_k \mathbf{H}_k)\mathbf{P}^j_{k|k-1}$ ▷ Updated state covariance
        end procedure
      end for
    end procedure
    procedure PARTICLE RE-SAMPLING($\{\xi^j_k\}$)
      Evaluate likelihood $\mathcal{L}(\xi^j_k) = \left((2\pi)^n \sqrt{|\mathbf{S}_k|} \right)^{-1} e^{-0.5 \epsilon^{j\,T}_k \mathbf{S}^{-1}_k \epsilon^j_k}$. ▷ For each ξ^j_k
      Normalized weight $w(\xi^j_k) = \frac{w(\xi^j_{k-1})\mathcal{L}\xi^j_k}{\sum_{j=1}^{N_p} w(\xi^j_{k-1})\mathcal{L}(\xi^j_k)}$ ▷ Updated weight
      Update: ▷ As per their weighted mean
      State $\mathbf{x}_{k|k} = \sum_{j=1}^{N_p} w(\xi^j_k)\mathbf{x}^j_{k|k}$ and Parameter estimates $\boldsymbol{\theta}_{k|k} = \sum_{j=1}^{N_p} w(\xi^j_k)\xi^j_k$
    end procedure
  end for
end procedure

4. NUMERICAL ANALYSIS AND DISCUSSION

Prior to employing the proposed spring-based model for defining the crack, it needs to be validated against real data. For this, the works of [Lee(2009)] has been taken basis on wherein certain experiments on cracked beam has been performed and the pertinent frequency domain information is disseminated.

4.1. *Model validation*

Following the article of [Lee(2009)], a cantilever beam, with material properties as: elastic modulus = $181\ GPa$, density = $7860\ kg/m^3$ and geometric properties as: span = $0.8\ m$, and area = $0.02 \times 0.02\ m^2$ is modelled. The beam has two cracks, with depth (**a**) and location (**c**) from the left end of the beam are $a_1 = 0.004\ m$, $a_2 = 0.006\ m$, $c_{l1} = 0.255\ m$ and $c_{l2} = 0.545\ m$. Subsequently, these cracks are replicated with rotational springs as mentioned before. The schematic of the modelling is presented in Figure 2.



(a) Actual beam with two edge cracks (b) Cracks modelled as mass-less spring

Figure 2. Schematic diagram of the modelling approach

First three natural frequencies of cantilever beam extracted from the proposed numerical model with cracks as rotational springs are compared with the experimental result as presented in [Lee(2009)]). Two edge cracks causing of 20% and 30% damage is equivalent to stiffness reduction factors of 20 and 17.5 in the springs considered as cracks and the comparison is presented in Table I demonstrating that such modelling strategy can safely be employed for model-based damage detection approach.

Table I. Comparison of first three frequencies of beam.

Undamaged Beam			Cracked Beam		
Experimental (Hz)	Numerical (Hz)	Error	Experimental (Hz)	Numerical (Hz)	Error
24.18	24.22	0.18	24.04	24.13	0.36
152.10	151.81	0.19	149.27	150.96	1.13
424.46	425.09	0.15	409.29	420.40	2.71

4.2. *Proposed filtering-based damage detection approach*

For the validation of the IPKF-based estimation approach, a simply supported EBB is adopted to represent a bridge structure wherein the soffit is induced with a crack. The geometry properties beam element are span = $1\ m$, and area = $0.02 \times 0.02\ m^2$ and material properties of beam element are elastic modulus = 200 GPa, density = 7850 kg/m^3 . The beam span is divided into 25 equal-length 2D EBB each representing a discrete element of the corresponding FEM. The 5^{th} element is induced with a numerical crack (cf.figure 3) by introducing a rotational spring of $\gamma = 2.5$ placed $0.02m$ from the left end of the element.

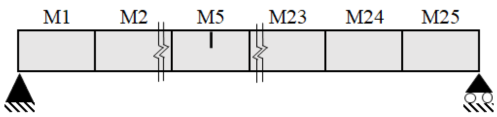


Figure 3. Schematic diagram of simply supported beam with single edge crack

Accordingly, the stiffness and mass matrices are obtained following the works of [Aswal et al.(2022)Aswal, Sen, and Mevel] taking the crack in to consideration as rotational spring. Strain response is simulated under an SWGN forcing and further contaminated with a 1% sensor noise (cf. Figure 4). Further, IPKF is employed for identification of crack features (cracked element, γ and c). In this algorithm (cf. algorithm 1), Kalman filter is used for state estimation and the estimated states are further used to reconstruct the strain measurement. A comparison between measured and estimated strain is reported in Figure 4 which shows a close match between these two entities.

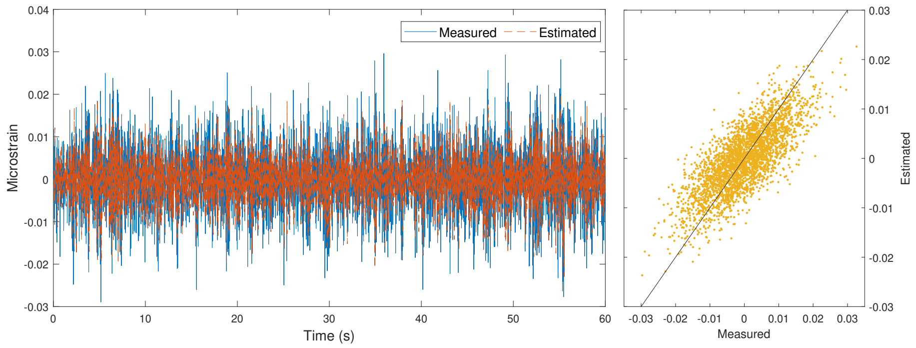
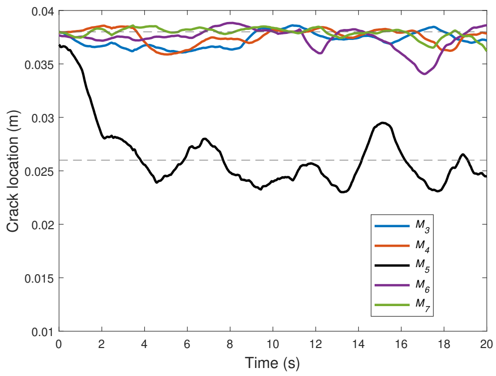
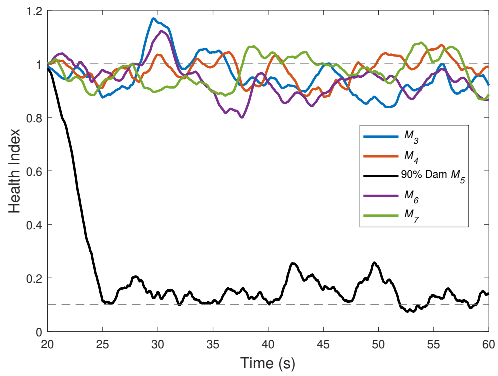


Figure 4. Comparison of strain measurement recreating from the estimated states to the actual strain

The damage features estimated with PF assuming 2000 particle to define the particle set. Starting from an arbitrary guess for the initial parameter set, re-sampling becomes necessary to avoid degeneracy. For the initialisation, c_l and γ are arbitrarily chosen as 0.038 m and 25 for each element and the estimated result is reported in the 5a and 5b. The results can be observed to be converging to their true values promptly not causing any false alarm.



(a) Parameter c_l estimation



(b) Parameter Health Index (HI) estimation

Figure 5. Performance of the proposed algorithm.(*dashed lines represent respective actual values*)

5. CONCLUSION

In this study, the crack is modelled as a mass-less rotational spring and the reduction in sectional rigidity due to the crack is defined through the reduction in the rotational stiffness of the spring. Prior to using this model type within a model-based crack detection algorithm powered by interacting Bayesian filter (IPKF), the adopted model is first validated against published results wherein real experiments are performed on cracked beam. The responses of the spring-based model is compared to that of the real experiments in frequency domain and convincing similarity is observed assuring such model can satisfactorily be used for crack detection algorithms. It should be noted that the current study do not distribute the effect of a localized damage like crack over the entire discretized element. Instead, the adopted model impacts the model right where the crack is. Detection of such localized damage otherwise demands a fine discretization rendering the adopted model to be computationally expensive and impractical to be integrated with a recursive real-time algorithm.

Current proposal adopts a two step approach for the detection wherein the damaged element is first isolated following a continuous variable denoting the crack location with respect to the damaged element is estimated along with the damage severity as the spring stiffness. This allows the detection to be independent of model discretization resolution, perfect for high dimensional structures. Numerical validation on a simply supported beam establishes the efficacy of the proposed approach.

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