# Subdomain boundary forces estimation of bridge structure under vehicle loading using interacting Ensemble-Particle filtering

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ABSTRACT

Vehicle-induced vibration may cause fatigue in bridge structures leading to sudden failure causing loss of economy and human lives. Structural fatigue estimation is however a complicated proposition since the entire structure needs to be monitored irrespective of the fatigue proneness of different parts of it. This eventually invites huge computational cost due to high dimensional modelling and also dense instrumentation for precise estimation of the fatigue related properties or parameters. It has been perceived that only a certain specific locations are fatigue prone and standalone fatigue monitoring of only that subdomain is possible provided the forces acting on to it is available. To isolate the subdomain of interest from the entire structural domain, one needs to compensate with boundary forces at the domain boundaries which is never possible to be measured. This study proposes a subdomain estimation based approach powered by interacting Ensemble-Particle filtering approach (IP-EnKF) that monitors only a subdomain of interest (fatigue prone) independently for its structural and damping properties while estimating the boundary forces in parallel for subsequent fatigue life estimation. This allows employment of computationally inexpensive predictor models for any model assisted health monitoring approach while reducing the required cost and effort of instrumentation. The approach has been validated on a bridge structure excited by a 4-degrees-of-freedom half-car model and subsequently unknown boundary forces are estimated for the fatigue life assessment. The proposed method estimates the boundary force under vehicle-bridge interaction has been found effective and reliable.

**Keywords**: subdomain, vehicle, boundary forces, particle filter, interacting filtering.

## 1. INTRODUCTION

In a fatigue failure scenario, bridges are subjected to continuous cyclic loads of varying magnitudes caused primarily by the passage of vehicles, which initiate cracks that propagate in the material. Interestingly, such failure does not require extreme loading on the structure. Instead, repeated small intensity loads can fail the structure without notice once the accumulated damage reaches its critical level [Antaki and Gilada(2015)]. Eventually, such failure poses major threats to civil infrastructure over their prolonged usage risking their stability and integrity. To avert that, fatigue life prediction (or Remaining Useful Life (RUL)) is crucial for bridges subjected to repeating uncertain dynamic loading. The uncertainty involved in such methods can be better dealt with probabilistic approaches [Kuncham et al.(2022)Kuncham, Sen, Kumar, and Pathak] which however demands the information about the stochastic force to be known. With a high dimensional structure like bridge, such expectations can never be satiated.

Thankfully, there are only a few locations in an entire bridge that can be identified as fatigue prone and needs focused investigation. Estimation of force statistics of that particular domain can therefore be more practical for the RUL prediction of the entire bridge. This way while only a subdomain of interest can be investigated more rigorously, the associated computational expenses can also be reduced by a scale. Such approaches are typically known as substructure estimation wherein the concerned subdomain is numerically isolated from the main structure complemented with a set of boundary forces. This study attempts to estimate the subdomain using an Interacting Bayesian filtering approach [Sen et al.(2021)Sen, Aswal, Zhang, and Mevel] while being robust to the boundary forces. Following, the boundary forces are reconstructed for further RUL estimation.

A recursive Bayesian framework naturally transforms state parameter estimation into a nonlinear problem, even when the dynamics of the system are linear [Tatsis et al.(2022)Tatsis, Agathos, Chatzi, and Dertimanis]. The nonlinearities in filter distributions make it impossible to evaluate them analytically, leading to sample based filter types like Particle Filters (PF) and Ensemble Kalman filter (EnKF). PF, although is a robust approach for nonlinear system estimation, introduces high computational cost [Chatzi and Smyth(2009)]. For a state parameter estimation problem, it is therefore more practical to employ an interacting PF-EnKF approach wherein PF estimates the parameter while the state estimation can be handled via EnKF [Hommels et al.(2009)Hommels, Murakami, and Nishimura].

The RUL estimation for a bridge depends on the force acting on it. This RUL estimate helps the owner of the infrastructure to take an informed decision on the future operation in terms of allowable traffic load and frequency, maintenance schedule etc. A model based approach for RUL estimation while can be validated under arbitrary loading scenario [Kuncham et al.(2022)Kuncham, Sen, Kumar, and Pathak], the central idea of connecting the RUL to the vehicle loading or frequency will surely be missed. This study therefore includes vehicle structure interaction that brings in the connection between traffic loading and the forces on the structure.

Simulating dynamic interactions between bridges and vehicles can be done in two ways. First, there is the uncoupled iteration method [Fafard et al.(1997)Fafard, Bennur, and Savard], which solves each system independently and performs iterative calculations at each time step in order to find the equilibrium between bridges and tires. The other method of simulating bridge-vehicle interaction involves solving the super-system fully coupled and obtaining the solution at each time-step without iteration. There are two different ways of modelling vehicle dynamics. First, there are the moving load models that use an influence line and dynamic amplification factor to model vehicle dynamics. However, such static model fails to take the influence of vehicle motion on bridge dynamics [Wang et al.(2020)Wang, Zhang, Tu, Sabourova, Grip, Blanksvärd, and Elfgren] into account. Alternatively vehicle models can be simulated by using a moving spring-mass-damper (MSMD) which is well-suited for modelling complex vehicle models. In moving vehicles, the load position changes with time, and the vehicle suspension oscillates due to irregularities in the bridge deck and vertical displacement of the bridge under tires [Duan and Yang(2013)]. These are the aspects which need consideration in the MSMD model. A MSMD coupled system was used by [Liu et al.(2013)Liu, Zhou, Shi, Wang, Shi, and De Roeck] to determine fatigue stresses in bridge structures.

The global objective of the research in progress, is to provide an RUL estimate for only the fatigue prone domain of a bridge and relate that to the fatigue loading and/or frequency. In achieving that, the current study proposes a computationally efficient force estimation approach for the substructure boundary wherein the boundary forces are simulated solving coupled dynamics of vehicle, structure and their interactions. The proposal attempts output injection methodology [Zhang and Zhang(2018)] to induce robustness in estimation against boundary forces which are then reconstructed for RUL estimation. For estimation, an interacting filtering strategy combining PF and EnKF (IPEnKF) was employed. The proposed algorithm has been validated numerically on a simply supported bridge structure excited by a half-car model with four degrees of freedom.

## 2. BRIDGE-VEHICLE INTERACTION

The dynamic equations of the half-car model vehicle [Duan and Yang(2013)] can be defined with mass $\mathbf{M}_v$, stiffness $\mathbf{K}_v$, damping $\mathbf{C}_v$ matrices and exciting force of vibration $\mathbf{F}_v$ as

$$\mathbf{M}_v\ddot{\mathbf{y}}_v + \mathbf{C}_v\dot{\mathbf{y}}_v + \mathbf{K}_v\mathbf{y}_v = \mathbf{F}_v \tag{1}$$

$$\mathbf{M}_v=\begin{bmatrix} m_s & 0 & 0 & 0\\ 0 & J & 0 & 0\\ 0 & 0 & m_{t1} & 0\\ 0 & 0 & 0 & m_{t2}\end{bmatrix},\ \mathbf{C}_v=\begin{bmatrix} c_{s1}+c_{s2} & c_{s1}a_1-c_{s2}a_2 & -c_{s1} & -c_{s2}\\ c_{s1}a_1-c_{s2}a_2 & c_{s1}{a_1}^2+c_{s2}{a_2}^2 & -c_{s1}a_1 & c_{s2}a_2\\ -c_{s1} & -c_{s1}a_1 & c_{s1}+c_{t1} & 0\\ -c_{s2} & c_{s2}a_2 & 0 & c_{s2}+c_{t2}\end{bmatrix},$$

$$\mathbf{K}_v=\begin{bmatrix} k_{s1}+k_{s2} & k_{s1}a_1-k_{s2}a_2 & -k_{s1} & -k_{s2}\\ k_{s1}a_1-k_{s2}a_2 & k_{s1}{a_1}^2+k_{s2}{a_2}^2 & -k_{s1}a_1 & k_{s2}a_2\\ -k_{s1} & -k_{s1}a_1 & k_{s1}+k_{t1} & 0\\ -k_{s2} & k_{s2}a_2 & 0 & k_{s2}+k_{t2}\end{bmatrix},\ \mathbf{F}_v=\begin{Bmatrix} 0\\ 0\\ k_{t1}y_{c1}+c_{t1}\dot{y}_{c1}\\ k_{t2}y_{c2}+c_{t2}\dot{y}_{c2}\end{Bmatrix},$$

$$\mathbf{y}_v=\{y_s \quad \theta \quad y_{t1} \quad y_{t2}\}^T$$

Where, $m_s$ is the mass of the body and frame of the vehicle (c.f Figure 1), $m_{t1}, m_{t2}$ are the mass of the axle between the front and back wheel set and the tires, $k_{s1}, k_{s2}, c_{s1}, c_{s2}$ are the stiffness and damping between wheel set and body of the vehicle, $k_{t1}, k_{t2}, c_{t1}, c_{t2}$ are the stiffness and damping between of the tires, $a_1, a_2$ are the displacement from the center of gravity to the back/front wheel set, $y_{c1}, y_{c2}$ are the displacement on the point which the bridge contacts with the front and back wheel set in the vertical, $\dot{y}_{c1}, \dot{y}_{c2}$ are the velocity at the point which the bridge contacts with the front and back wheel set.

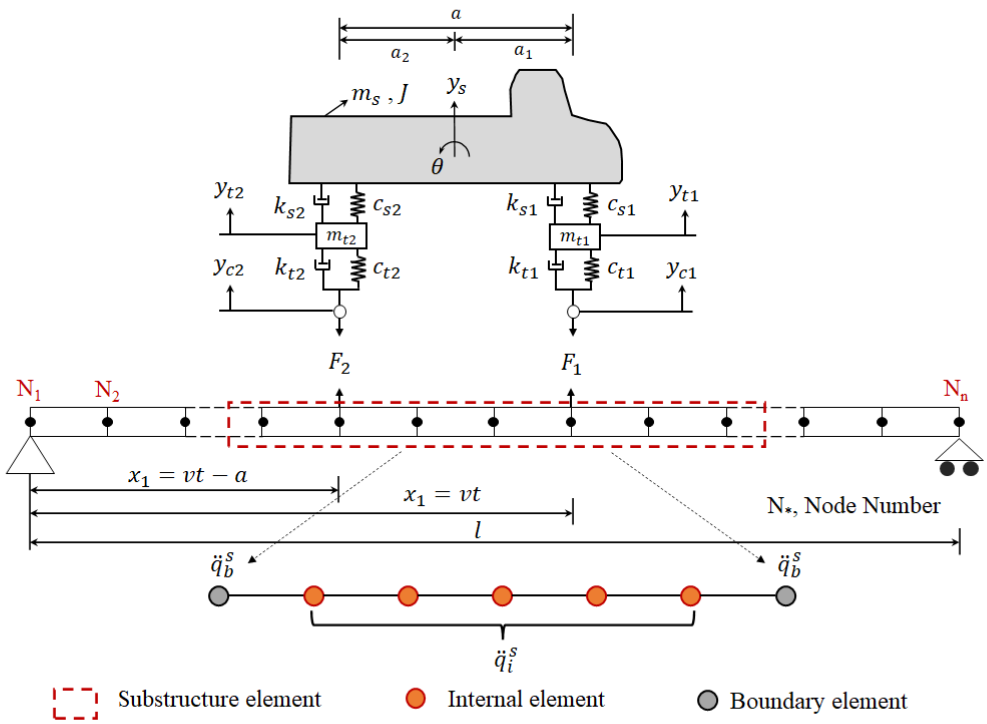



Figure 1. Recreating the boundary response from the estimated states

The modal equation for the bridge under half car model can be expressed as,

$$\ddot{\eta}_n+2\zeta_n\omega_n\dot{\eta}_n+\omega_n^2\eta_n=-F_1(t)\phi_{1n}\delta_1-F_2(t)\phi_{2n}\delta_2 \tag{2}$$

$$\phi_{1n}=\sqrt{\frac{2}{\rho l}}sin\frac{n\pi vt}{l},\ \phi_{2n}=\sqrt{\frac{2}{\rho l}}sin\frac{n\pi(vt-a)}{l},\ \delta_1(t)=\begin{cases}1, 0\le t\le \frac{l}{v}\\ 0, else\end{cases},\ \delta_2(t)=\begin{cases}1, \frac{a}{v}\le t\le \frac{l+a}{v}\\ 0, else\end{cases}$$

Here, $2\zeta_n\omega_n=\frac{\mu}{\rho}, \omega_n^2=\frac{EI}{\rho}(\frac{n\pi}{l})^4$, $E$ is Young's modulus of the bridge, $I$ is the moment of inertia of the cross section, $\rho$ is the mass of the bridge per unit length, $\mu$ is the damping coefficient per unit length $(l)$, $F$ is the vehicle-bridge coupled force on the bridge, $\eta$ is modal coordinates of the bridge, $n$ is the number of modes considered for the vibration analysis and $v$ is the velocity of the vehicle.

Based on the compatibility condition, vehicle-bridge coupled function has been computed as

$$\frac{a_2\phi_{1n}\delta_1 + a_1\phi_{2n}\delta_2}{a} m_s\ddot{y}_s + \frac{\phi_{1n}\delta_1 - \phi_{2n}\delta_2}{a} J\ddot{\theta} + \phi_{1n}\delta_1 m_{t1}\ddot{y}_{t1} + \phi_{2n}\delta_2 m_{t2}\ddot{y}_{t2} + \\ \ddot{\eta}_n + 2\zeta_n\omega_n\dot{\eta}_n + \omega_n^2\eta_n = -[\phi_{1n}W_1\delta_1 + \phi_{2n}W_2\delta_2] \quad (3)$$

where, $W_i$ is the static load acting on the wheel due to the vehicle weight. $W_1 = (m_s\frac{a_2}{a} + m_{t1})g$, $W_2 = (m_s\frac{a_1}{a} + m_{t2})g$ and $g$ is the acceleration due to gravity. Further converting Equation 3 to the standard dynamic equation of the coupled vehicle-bridge can express as follows

$$\mathbf{M}(t)\ddot{\mathbf{U}}(t) + \mathbf{C}(t)\dot{\mathbf{U}}(t) + \mathbf{K}(t)\mathbf{U}(t) = \mathbf{Q}(t) \quad (4)$$

here mass $\mathbf{M}(t)$, stiffness $\mathbf{K}(t)$, damping $\mathbf{C}(t)$ matrices for the coupled system with degrees of freedom as $\mathbf{U} = \begin{Bmatrix} y_s & \theta & y_{t1} & y_{t2} & \eta_1 & \eta_2 & \cdots & \eta_n \end{Bmatrix}^T$ and $\mathbf{Q}(t)$ is the force on the coupled system. For detail derivation and corresponding matrices the reader may follow [Duan and Yang(2013)]. Using the mode shape matrix of the bridge structure, modal domain response $\mathbf{U}$ can further be converted to time domain response, denoted as $\mathbf{q}$. Accordingly, $\ddot{\mathbf{q}}$, i.e. bridge acceleration responses are collected as measurements from where inference can be drawn during RUL estimation. Eventually, by analysing the responses the fatigue prone area can be identified which can further be isolated as a substructure in order to estimate the forces acting on its boundaries.

## 3. SUBSTRUCTURE SYSTEM FORMULATION FOR BOUNDARY FORCE ESTIMATION

The governing dynamic equation of a bridge substructure $\Omega^s$ can be defined as [Aswal et al.(2023)Aswal, Sen, and Mevel],

$$\dot{\mathbf{x}}^s(t) = \mathbf{F}^s\mathbf{x}^s(t) + \mathbf{B}^s\mathbf{u}^s(t) + \mathbf{E}^s\ddot{\mathbf{q}}_b^s(t) + \mathbf{v}^s(t) \quad (5)$$

where, $\dot{\mathbf{x}}^s(t) = \begin{Bmatrix} \mathbf{q}_i{}^{s,r} \\ \dot{\mathbf{q}}_i{}^{s,r} \end{Bmatrix}$, $\mathbf{F}^s = \begin{bmatrix} \mathbf{0}_{n_i} & \mathbf{I}_{n_i} \\ -\mathbf{M}_{ii}^{s\,-1}\mathbf{K}_{ii}^s & -\mathbf{M}_{ii}^{s\,-1}\mathbf{C}_{ii}^s \end{bmatrix}$, $\mathbf{B}^s = \begin{bmatrix} \mathbf{0}_{n_i} \\ \mathbf{M}_{ii}^{s\,-1} \end{bmatrix}$, $\mathbf{u}^s(t) = \mathbf{F}_{bv}^{int}$ and $\mathbf{E}^s = \begin{bmatrix} \mathbf{0}_{n_i} \\ -(\mathbf{M}_{ii}^{s\,-1}\mathbf{M}_{ib}^s + \eta^s) \end{bmatrix}$. $\mathbf{M}, \mathbf{K}$, and $\mathbf{C}$ denotes mass, stiffness and damping matrices of $\Omega^s$. Subscript $i$ and $b$ signifies internal and boundary degrees of freedom (*dof*s) and superscript $s$ denotes the association to substructure $\Omega^s$. $\mathbf{F}_{bv}^{int}$ denotes the force acting on the internal nodes of $\Omega^s$ while $\ddot{\mathbf{q}}_b^s$ is the force acting on its boundaries. $\eta^s$ is acting as a transmissibility term correlating boundary to internal responses. Please refer [Aswal et al.(2023)Aswal, Sen, and Mevel] for detail derivation. $\mathbf{v}^s(t)$ represents process uncertainty originating from the model inaccuracies and unmodelled inputs. The measurable acceleration responses $\ddot{\mathbf{q}}_i^s$ correspond to total acceleration due to pseudo-static ($\ddot{\mathbf{q}}_b^{s,d}$) and relative ($\ddot{\mathbf{q}}_b^{s,r}$) response components combined as $\mathbf{y}^s(t)$. Accordingly,

$$\mathbf{y}^s(t) = \ddot{\mathbf{q}}_i^{s,r}(t) + \eta^s\ddot{\mathbf{q}}_b^s(t) = S\{\mathbf{H}^s\mathbf{x}^s(t) + \mathbf{D}^s\mathbf{u}^s(t) + \mathbf{L}^s\ddot{\mathbf{q}}_b^s(t) + \mathbf{w}^s(t)\} \quad (6)$$

here, $\mathbf{H}^s = \begin{bmatrix} -\mathbf{M}_{ii}^{s\,-1}\mathbf{K}_{ii}^s & -\mathbf{M}_{ii}^{s\,-1}\mathbf{C}_{ii}^s \end{bmatrix}$, $\mathbf{D}^s = \mathbf{M}_{ii}^{s\,-1}$, and $\mathbf{L}^s = -\mathbf{M}_{ii}^{s\,-1}\mathbf{M}_{ib}^s$ and $\mathbf{w}^s(t)$ denoting measurement noise. S represents the Boolean selection matrix defining the measured *dof*s. Since in reality, responses are discretely sampled, Equations (2) and (3) can also be presented in discrete time with continuous variables are reproduced with their corresponding discrete time entities.

$$\begin{aligned} \mathbf{x}_k^s &= \mathbf{F}_k^s\mathbf{x}_{k-1}^s + \mathbf{B}_k^s\mathbf{u}_k^s + \mathbf{E}_k^s\ddot{\mathbf{q}}_{b,k}^s + \mathbf{v}_k^s \\ \mathbf{y}_k^s &= \mathbf{H}_k^s\mathbf{x}_k^s + \mathbf{D}_k^s\mathbf{u}_k^s + \mathbf{L}_k^s\ddot{\mathbf{q}}_{b,k}^s + \mathbf{w}_k^s = \mathcal{Z}(\mathbf{x}_k^s, \ddot{\mathbf{q}}_{b,k}^s) \end{aligned} \quad (7)$$

In order to eliminate the interface forces from the state equation, output injection technique [Zhang and Zhang(2018)] can further be approached wherein the output is injected through a full rank matrix $\mathbf{G}_k^s$ 4. Accordingly, the state equation gets modified as,

$$\mathbf{x}_k^s = \widetilde{F}_k^s \mathbf{x}_{k-1}^s + \widetilde{B}_k^s \mathbf{u}_k^s + \widetilde{E}_k^s \ddot{\mathbf{q}_{b,k}}^s + \mathbf{G}_k^s \mathbf{y}_k^s + \widetilde{\mathbf{v}}_k^s \tag{8}$$

With $\mathcal{L}_k^s = \mathbf{I} - \mathbf{G}_k^s \mathbf{H}_k^s$, $\widetilde{F}_k^s = \mathcal{L}_k^s \mathbf{F}_k^s$, $\widetilde{B}_k^s = \mathcal{L}_k^s \mathbf{B}_k^s - \mathbf{G}_k^s \mathbf{D}_k^s$, $\widetilde{E}_k^s = \mathcal{L}_k^s \mathbf{E}_k^s - \mathbf{G}_k^s \mathbf{L}_k^s$, and $\widetilde{\mathbf{v}}_k^s = \mathbf{v}_k^s - \mathbf{G}_k^s \mathbf{w}_k^s$. If $\mathbf{G}_k^s$ is chosen such that $\mathbf{G}_k^s = \mathbf{E}_k^s(\mathbf{H}_k^s \mathbf{E}_k^s + \mathbf{L}_k^s)^\dagger$ with † denoting Moore-Penrose Pseudo-inverse operation, $\widetilde{E}_k^s$ renders to a null matrix. Equation (4) can then be transformed to Equation (5), decoupled from the boundary measurements as $\ddot{\mathbf{q}}_{b,k}^s$,

$$\mathbf{x}_k^s = \widetilde{F}_k^s \mathbf{x}_{k-1}^s + \widetilde{B}_k^s \mathbf{u}_k^s + \mathbf{G}_k^s \mathbf{y}_k^s + \widetilde{\mathbf{v}}_k^s = \mathcal{F}_1(\mathbf{x}_{k-1}^s, \mathbf{y}_k^s, \widetilde{\mathbf{v}}_k^s) \tag{9}$$

Thus, with Bayesian filtering approaches, subdomain $\Omega^s$ can be estimated without measuring the interface response. In this study, to address the real life complexity, the stiffness parameters of the bridge has been assumed to be unknown and therefore estimated in parallel to the system states $\mathbf{x}_k^s$. This necessitates the application of the mentioned IP-EnKF algorithm which decouples the estimation of states and parameters and employ separate filters for each. Finally, with the system states estimated drawing inference from measurements, boundary force can then be reconstructed as,

$$\ddot{\mathbf{q}}_{b,k}^s = [\ \widetilde{E}_k^s\ ]^{-1} [\ \mathbf{x}_k^s - (\widetilde{F}_k^s \mathbf{x}_{k-1}^s + \widetilde{B}_k^s \mathbf{u}_k^s + \mathbf{G}_k^s \mathbf{y}_k^s + \widetilde{\mathbf{v}}_k^s)\ ] = \mathcal{F}_2(\mathbf{x}_{k-1}^s, \mathbf{x}_k^s, \mathbf{y}_k^s, \widetilde{\mathbf{v}}_k^s) \tag{10}$$

The boundary force is then estimated by applying the inertial force formation as,

$$\ddot{\mathbf{F}}_{b,k}^s = \mathbf{M}_{bb} \ddot{\mathbf{q}}_{b,k}^s \tag{11}$$

A pseudo-code of the proposal is presented in Algorithm 1.

## 4. NUMERICAL STUDY

To validate the proposed algorithm, dynamic analysis has been performed on a simply supported box-girder bridge subjected to half car moving vehicle at constant speed (V) of 20 $kmph$. Details of bridge and vehicle model are given in Table I. The bridge span of length $l = 60\ m$ is divided into 20 equal parts as elements which are further defined with two-node 4$dof$s Euler Bernoulli beam model. Following, the fatigue prone zone in the bridge span is identified and the proposed algorithm is employed considering this domain as the monitored substructure (cf. Figure 1). The assumed parameters for the numerical simulation are given in Table I.

Table I. Bridge and vehicle parameters for numerical study

| | | | | | |
|---|---|---|---|---|---|
| **Parameters of the bridge** | | | | | |
| EI $(N-m^2)$ | 2.12E11 | $\rho\ (kg/m)$ | 3.03E+04 | $\mu$ | 0.02 |
| **Parameters of the vehicle** | | | | | |
| $m_s\ (kg)$ | 38500 | J $(kgm^2)$ | $2.446 \times 10^6$ | $m_{t1}(kg)$ | 4330 |
| $m_{t2}(kg)$ | 4330 | $k_{s1}(N/m)$ | $2.535 \times 10^6$ | $k_{s2}(N/m)$ | $2.535 \times 10^6$ |
| $k_{t1}(N/m)$ | $4.28 \times 10^6$ | $k_{t2}\ (N/m)$ | $4.28 \times 10^6$ | $c_{s1}$ (N/sm) | $1.96 \times 10^5$ |
| $c_{s2}(N/sm)$ | $1.96 \times 10^5$ | $c_{t1}(N/sm)$ | $9.8 \times 10^4$ | $c_{t2}(N/sm)$ | $9.8 \times 10^4$ |
| $a_1(m)$ | 4.2 | $a_2(m)$ | 4.2 | $V(kmph)$ | 20 |

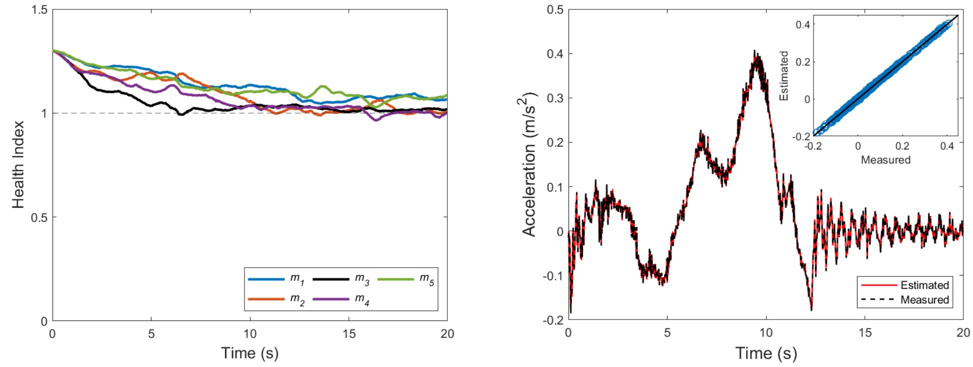
The bridge structure is excited under aforementioned vehicle loading, and acceleration responses are sampled from the chosen substructure nodes (internal $dof$s only) at a sampling frequency of 50 Hz for 20 seconds under its undamaged state. The simulated signal is further contaminated with stationary white Gaussian noise (SWGN) having a signal-to-noise ratio of $1\%$ in order to emulate real-life situations. Initial distribution of the parameters is assumed to be $\mathcal{N}(1.3, 0.01)$, and $\alpha$ is set

**Algorithm 1** Proposed boundary force estimation algorithm for substructures

**procedure** IP-ENKF($\mathbf{y}_k, \mathbf{Q}, \mathbf{R}$) ▷ Interacting particle and Ensemble Kalman filter algorithm
    Initialize particles $\{\xi_0^j\}$, and state estimates $\{\mathbf{x}_{0|0}^{i,j}\}$ ▷ Initial values
    **for** <each $k^{th}$ measurement $\mathbf{y}_k$> **do**
        **procedure** IP-ENKF($\{\xi_{k-1}^j\}, \{\mathbf{x}_{k-1|k-1}^{i,j}\}$) ▷ Initiating Ensemble Kalman filter
            **for** <each particle $\xi_k^j$> **do** ▷ Initiating particle filter
                Evolve $\{\xi_{k-1}^j\} \rightarrow \{\xi_k^j\}$
                **procedure** ENKF($\xi_k^j, \{\mathbf{x}_{k-1|k-1}^{i,j}\}, \mathbf{y}_k$) ▷ For each $j^{th}$ particle
                    **for** <each ensemble $\mathbf{x}_{k-1|k-1}^{i,j}$> **do**
                        **Prediction**: Propagate state to $\mathbf{x}_{k|k-1}^{i,j}$ ▷ Equation (9)
                        Boundary measurement, $\ddot{\mathbf{q}}_{b,k|k-1}^{i,j}$ ▷ Equation (10)
                        Estimate measurement, $\mathbf{y}_{k|k-1}^{i,j}$ ▷ Equation (7)
                    **end for**
                    **Mean calculation:**
                    $\mathbf{x}_{k|k-1}^{j} = \mathbf{x}_{k|k-1}^{j} = \frac{1}{N_e}\sum_{i=1}^{N_e}\mathbf{x}_{k|k-1}^{i,j}$ ▷ Propagated state estimates
                    $\mathbf{y}_{i,k|k-1}^{j} = \frac{1}{N_e}\sum_{i=1}^{N_e}\mathbf{y}_{i,k|k-1}^{i,j}$ ▷ Estimated internal measurements
                    $\ddot{\mathbf{q}}_{b,k|k-1}^{j} = \frac{1}{N_e}\sum_{i=1}^{N_e}\ddot{\mathbf{q}}_{b,k|k-1}^{i,j}$ ▷ Propagated boundary measurements
                    $\varepsilon_k^j = \frac{1}{N_e}\sum_{i=1}^{N_e}\varepsilon_k^{i,j}$ where $\varepsilon_k^{i,j} = \mathbf{y}_k - \mathbf{y}_{k|k-1}^j$ ▷ Evaluate overall innovation
                    Estimated boundary force $\ddot{\mathbf{F}}_{b,k|k-1}^{j}$ ▷ Equation (11)
                    **Covariance calculation:**
                    $C_k^{j,xy} = \frac{1}{N_e-1}\sum_{i=1}^{N_e}\left(\mathbf{x}_{k|k-1}^{j} - \mathbf{x}_{k|k-1}^{i,j}\right)\left(\mathbf{y}_{k|k-1}^{j} - \mathbf{y}_{k|k-1}^{i,j}\right)^T$ ▷ Equation (9),(7)
                    $C_k^{j,yy} = \frac{1}{N_e-1}\sum_{i=1}^{N_e}\left(\mathbf{y}_{k|k-1}^{j} - \mathbf{y}_{k|k-1}^{i,j}\right)\left(\mathbf{y}_{k|k-1}^{j} - \mathbf{y}_{k|k-1}^{i,j}\right)^T$ ▷ Equation (7)
                    **Correction**: Innovation error: $\mathbf{S}_k^j = C_k^{j,yy} + \mathbf{R}$, & EnKF gain $\mathbf{G}_k^j = C_k^{j,xy}(\mathbf{S}_k^j)^{-1}$
                    $\mathbf{x}_{k|k}^{i,j} = \mathbf{x}_{k|k-1}^{i,j} + \mathbf{G}_k^j \in_k^{i,j}$ ▷ Correct predicted state estimate
                **end procedure**
                Calculate ensemble mean of the corrected state, i.e., $\mathbf{x}_{k|k}^j$
            **end for**
        **end procedure**
        **procedure** PARTICLE RE-SAMPLING($\{\xi_k^j\}$)
            Evaluate likelihood $\mathcal{L}(\xi_k^j) = \left((2\pi)^n\sqrt{|\mathbf{S}_k|}\right)^{-1} e^{-0.5 {\in_k^j}^T \mathbf{S}_k^{-1} \in_k^j}$. ▷ For each $\xi_k^j$
            Normalized weight $w(\xi_k^j) = \frac{w(\xi_{k-1}^j)\mathcal{L}\xi_k^j}{\sum_{j=1}^{N_p} w(\xi_{k-1}^j)\mathcal{L}(\xi_k^j)}$ ▷ Updated weight
            **Update**: ▷ As per their weighted mean
            State $\mathbf{x}_{k|k} = \sum_{j=1}^{N_p} w(\xi_k^j)\mathbf{x}_{k|k}^j$ and Parameter estimates $\boldsymbol{\theta}_{k|k} = \sum_{j=1}^{N_p} w(\xi_k^j)\xi_k^j$
        **end procedure**
    **end for**
**end procedure**

to 0.98. Numerical experiments were conducted using 2000 filter particles for PF and 100 ensembles for EnKF.

The proposed algorithm is validated under undamaged condition wherein the location based undamaged health parameters (normalized elasticity of each substructure elements) are considered to be unknown. The mean of initial estimate is considered to 1.3 which is further estimated to check if it converges to its true value, i.e. 1. In Figure 2a, smooth convergence of all five substructure element elasticity (normalized) to their true values can be verified. The comparison between estimated and actual (simulated) acceleration of an internal *dof* is presented in Figure 2b. Similarly, estimated, and actual boundary force (simulated) is compared in Figure 3. In both the cases, close matching can be observed through out the signal length which can also be verified from the scatter plots presented in additional.




(a) Estimation of health indices (*dashed lines represent respective actual values*)

(b) Comparison of internal response recreated from the estimated states

Figure 2. Performance of the proposed algorithm under vehicle loading.

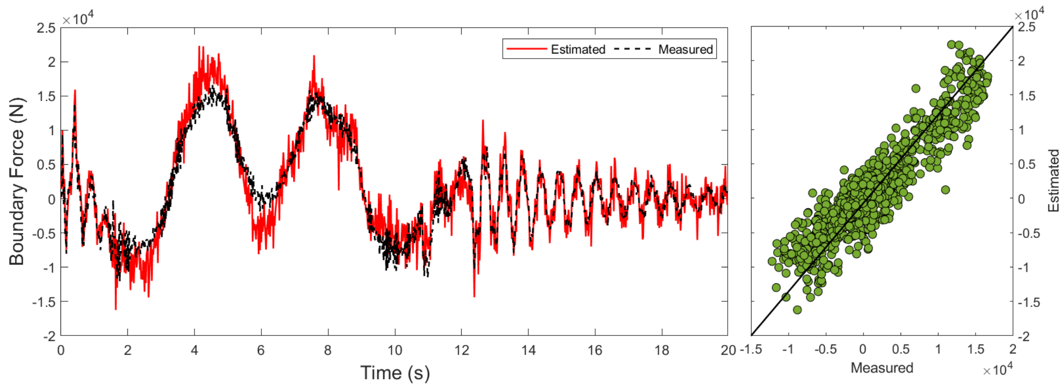



Figure 3. Comparison of boundary force recreating from the estimated states to the actual excitation

## 5. CONCLUSIONS

RUL of a bridge infrastructure can be predicted if information regrading structural loading and time varying material properties are available. In order to enhance the service life, while controlling the material properties is a difficult proposition for the owner, maintaining loading can be an option. However, the traffic loading has to be mapped to corresponding structural loading, which has been attempted in this article through vehicle structure interaction. Moreover, fatigue failures are typically limited to critical areas rather than the entire structural domain. Monitoring an entire structure in process of RUL estimation can be tedious job which can however be circumvented through substructure monitoring. Nevertheless, this demands the substructure interface to be either estimated or measured. In the context of fatigue life estimation, a Bayesian filtering based approach has been adopted wherein additionally, the force on the fatigue prone substructure is estimated. This study proposes an interacting filtering-based computationally efficient substructure boundary force estimation approach that can efficiently provide the information regarding structural loading and material parameters to be used for RUL estimation. The proposed algorithm has been validated with numerical experiments, and the results demonstrate its precision and accuracy in estimating boundary forces.

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