Flow Past a Rigid Sphere in a Couple Stress Fluid at Low Reynolds Numbers

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1. INTRODUCTION & OBJECTIVE

In this paper, I have attempted to bring the hydrodynamic force for the flow past the rigid spherical particle given by Lovalenti and Brady [1] and combine them with the drag described by Stokes [2]. In the next section I have given the mathematical formulation and in the third section I have shown the tests performed and see the subtle difference in the inclusion of couple stresses and the low Reynolds numbers of the fluid.

2. MATHEMATICAL FORMULATION

The formalism developed by Lovalenti and Brady [1] for the hydrodynamic force in the limit of low Reynolds numbers on a time dependent motion of a spherical particle along with the Newton' Law of motion used to combine the force expression of Stokes [2] to arrive at the expression for the net force on the spherical particle is given by,

$$\boldsymbol{F^H}(t) = ReSl\dot{\boldsymbol{U}}^{\infty}(t) - 6\pi a\boldsymbol{U_s}(t)\left(2+\frac{\alpha}{a}\right) - \frac{2\pi}{3}ReSl\dot{\boldsymbol{U}}_{\boldsymbol{s}}(t)$$

$$+\frac{3}{8}\left(\frac{ReSl}{\pi}\right)^{\frac{1}{2}}\left\{\int_{-\infty}^{t}\left\{\frac{2}{3}\boldsymbol{F}_{\boldsymbol{s}}^{\boldsymbol{H}_{\parallel}}(t) - \left[\frac{1}{|\boldsymbol{A}|^2}\left(\frac{\sqrt{\pi}}{2|\boldsymbol{A}|}erf(|\boldsymbol{A}|) - exp(-|\boldsymbol{A}|^2)\right)\right]\boldsymbol{F}_{\boldsymbol{s}}^{\boldsymbol{H}_{\parallel}}(s)\right.\right.$$

$$\left.(+\frac{2}{3}\boldsymbol{F}_{\boldsymbol{s}}^{\boldsymbol{H}_{\perp}}(t) - \left[exp(-|\boldsymbol{A}|^2) - \frac{1}{2|\boldsymbol{A}|^2}\left(\frac{\sqrt{\pi}}{2|\boldsymbol{A}|}erf(|\boldsymbol{A}|) - exp(-|\boldsymbol{A}|^2)\right)\right]\boldsymbol{F}_{\boldsymbol{s}}^{\boldsymbol{H}_{\perp}}(s)\right\}$$

$$\left.\frac{2ds}{(t-s)^{3/2}}\right\} + o(Re) \tag{1}$$

The meaning of the notations can be referred in [1] and [2].

3. RESULTS & HIGHLIGHTS OF IMPOINTANT POINTS

The program to solve the integral using OCTAVE were written and the results of Lovalenti and Brady [2] and Stokes' [4] were reproduced. Hence I claim that the programs are robust and valid. Further the difference between the inclusion of couple stresses is that there is a decay in the drag ratio when the radius of sphere increases as seen in figure (1), where as in the absence of couple stress the Reynolds number is attained as the particle velocity with respect to particle position as in figure(2). Further the inclusion of both leads to the convergence of the curves of the first figures. That is all the curves of the figure(1) irrespective of the μ value merge with the last curve which is due to the inclusion of resistance to the change in the change in the motion of the fluid.

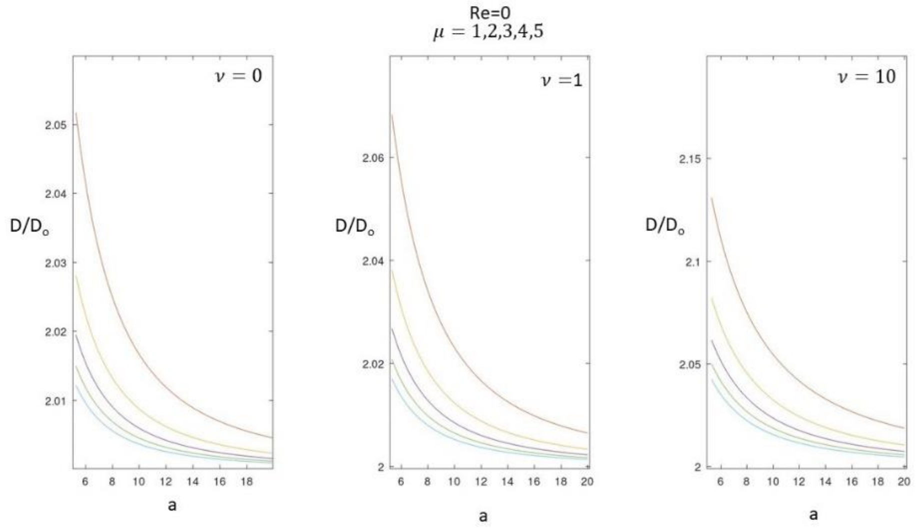
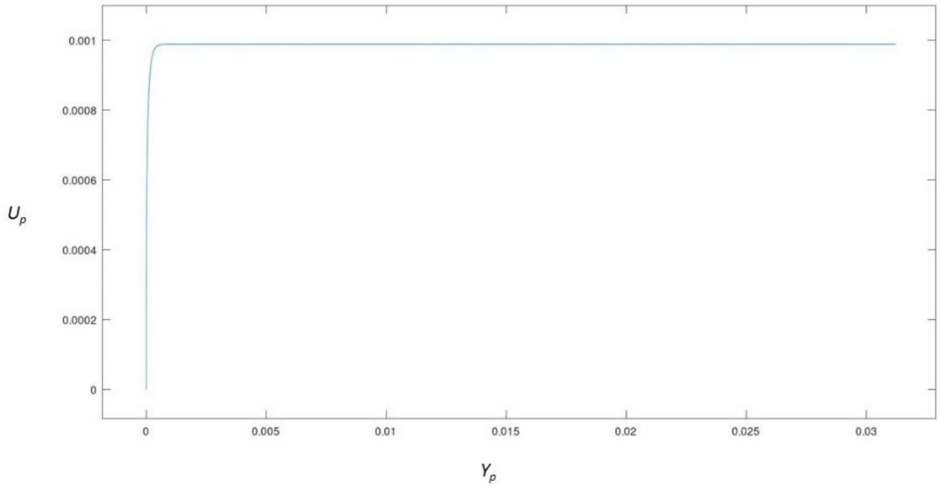


Figure (1): Drag Ratio with respect to radius of the sphere (a) for Re=0 and different μ and ν which is the replica of Stokes [2]



Figure(2): The phase plot reproduced of Lovalenti and Brady [1]

REFERENCES

1. Lovalenti PM and Brady JF, *The hydrodynamic force of a rigid particle undergoing arbitrary time dependent motion at small Reynolds numbers*, J Fluid Mech **256,** pp 561 – 605, 1993
2. V K Stokes, *Effects of Couple Stresses in fluids on a creeping flow past a sphere,* Phys. Fluids **14,** pp. 1580 – 1582, 1971.