Rotating Magnetoconvection with Anisotropic Magnetic Diffusivity – Weakly Nonlinear Analysis

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ABSTRACT

The anisotropy in the Earth's outer core strongly influences some convection processes very important in Core Dynamics. For instance, some instabilities in rotating magnetoconvection, described usually by the analysis in terms of normal modes, depend strictly on the anisotropic diffusion. Thus, many models concerning the stationary and oscillating modes of rotating magnetoconvection with different cases of anisotropy in the viscosity, thermal and magnetic diffusivities. In all cases, an anisotropy greater in the vertical direction parallel to gravity ("atmospheric anisotropy") facilitates the convection, while an anisotropy greater in horizontal directions ("oceanic anisotropy") inhibits some types of convection. The stability of a horizontal fluid planar layer rotating about its vertical axis and permeated by the horizontal homogeneous magnetic field is studied. The influence of anisotropic magnetic diffusive coefficient is analyzed in this work. In the vicinity of the primary instability threshold, the weakly nonlinear behavior of the convective motion is studied using the cubic nonlinear Landau-Ginzburg equation. This equation is obtained using the multiple scale analysis and by the modified normal mode method. The linear stability analysis of Landau-Ginzburg equation is performed for investigating the existence of Eckhaus instability.

LITERATURE SURVEY

In the geodynamo problem the basic Magnetic, Archimedean and Coriolis (MAC) dynamic can be sensitively affected by diffusion processes and weakening the individual forces. Thus the magnetic and thermal diffusion weaken the M and A forces, respectively, while viscosity weakens the C force, which can lead to a new balance of the forces with the possibility of instabilities that are completely different from those in the without diffusion. Despite the fact that some diffusion coefficients are unrealistically high, e.g. viscosity, due to numerical limitations, results of the simulations are of great heuristic significance (Fearn and Roberts, 2007; Gubbins and Herrero-Bervera, 2007). The effects of various anisotropic diffusion coefficients on the geodynamo or at least magnetoconvection are known. In the present study a simple model related to many geophysical and astrophysical flows, such as oceanic deep convection and the convective outer layer of the Sun, is considered, which is rotating Rayleigh–Bénard convection (RBC) under the influence of magnetic field. Such problems are suitable for conditions in the Earth's mantle. Donald and Roberts (2004) investigated the influence of anisotropic thermal diffusive coefficient (κ) in RBC under the influence of magnetic field. This problem is extended by Filippi and Brestenský (2020) by considering anisotropic β-effect in the influence of pure-*m* anisotropic diffusive coefficient (η) and partial q anisotropic diffusive coefficients (η, κ). Further the linear stability analysis of such problem has been studied by Jones and Roberts (2000); Donald and Roberts (2004); Šoltis and Brestenský (2010) and Filippi et. al. (2019) near the onset of convection. More results can be unearthed if the weakly nonlinear analysis is performed on the RMC along with oscillatory convection. Hence an attempt is made in the present study to analyze this established problem, near the convection, using the normal mode method related to the linear stability and the multiple scale analysis related to weakly nonlinear analysis. The linear stability analysis of the derived cubic nonlinear amplitude equation is performed to investigate the secondary instabilities, such as, Eckhaus instability.

PROBLEM STATEMENT

Two horizontal infinite plane layers filled with electrically conducting fluid kept in the homogeneous horizontal magnetic field that and rotating about its vertical axis is considered. The layers are heated from below and cooled from above. As in various settings, most notably in geophysical applications, material property η is assumed to be anisotropic and the corresponding perturbed governing equations are considered as below:

$$R_0\left[\frac{\partial\vec{u}}{\partial t}+\left(\vec{u}\cdot\vec{\nabla}\right)\vec{u}\right]+\hat{z}\times\vec{u}=-\vec{\nabla}P+\Lambda\left(\vec{b}\cdot\vec{\nabla}\right)\vec{b}+\Lambda\frac{\partial\vec{b}}{\partial y}+R\theta\hat{z}+E_z\nabla^2\vec{u}\,,$$

$$\frac{\partial\vec{b}}{\partial t}=\vec{\nabla}\times\left(\vec{u}\times\vec{b}\right)+\vec{\nabla}\times\left(\vec{u}\times\hat{y}\right)+\nabla_\alpha^2\vec{b},\frac{1}{q_z}\frac{\partial\theta}{\partial t}+\left(\vec{u}\cdot\vec{\nabla}\right)\theta=\nabla^2\theta,\vec{\nabla}\cdot\vec{u}=0,\vec{\nabla}\cdot\vec{b}=0\,.$$

where the notations are explained in Šoltis and Brestenský (2010) and R_0 = Modified Rossby Number, Λ =Elsasser Number, E_z =Ekman Number, R =Modified Rayleigh Number and q_z =Roberts Number. It can be noted that $\nabla_\alpha^2 \vec{b}$ in the magnetic induction equation takes care about the anisotropic magnetic diffusivity.

SOLUTION METHODOLOGY

The above non-dimensional perturbed governing equations have been converted to a single equation by taking the z-component of the curl and double curl of the Navier-Stokes equation, the curl of the induction equation and induction equation itself and the equation of heat conduction. The linear stability analysis has been performed using normal mode analysis. The critical R and critical wavenumber have been derived for the case of Oblique rolls, Normal rolls, and Cross rolls with different values of E_z and Λ. The weakly nonlinear stability analysis has been investigated using multiple scale analysis and the Landau-Ginzburg equation is derived. The heat transfer rate has been calculated. The region of Eckhaus instability has been shown for the different values of E_z, Λ, R_o and q_z.

CONCLUSIONS

The influence of anisotropic thermal diffusive coefficient on the marginal and weakly nonlinear stability near the onset of convection of a horizontal planar layer of electrically conducting fluid has been studied. The layer is permeated by a horizontal homogeneous magnetic field and rotates about a vertical axis of rotation. The present study is advanced by using weakly nonlinear analysis with the thermal diffusive anisotropy of stratification anisotropy *(SA)*. It is observed that in the case of parallel rolls for every E_z, the critical R does not depend on the Λ. When the E_z decreases (i.e. low to high rotation) the heat transfer rate (Nu) increases for all R values with fixed values of Λ, R_o and q_z. For Sun and in the Earth's core R_o is very small. If R_o decreases 10^{-5} to 10^{-15} (i.e. high to low magnetic diffusion) then the Nu increases for all R by fixing E_z at 10^{-5} (i.e. at high rotation), R_o and Λ as 1. When the E_z decreases (i.e. low rotation to high rotation) and R_o decreases (i.e. high to low magnetic diffusion) the nonlinear coefficient of the amplitude equation decreases in the supercritical region for both isotropic and anisotropic cases.

References

1. Jones, C.A., and Roberts, P.H., *"The onset of magnetoconvection at large Pandtl number in a rotating layer. II. Small magnetic diffusion"*, Geophys. Astrophys. Fluid Dyn. 2000, 93, 173-226.
2. Y. Rameshwar, G. Srnivas, A. Krishna Rao, U. S. Mahabaleshwar and D. Laroze, *"Nonlinear Analysis of Cross Rolls of Electrically Conducting Fluid under an Applied Magnetic Field with Rotation"*, Processes, 2023, (11) 1-30.
3. Filippi, E. and Brestenský, J., *"Anisotropic β-effect in the diffusive processes in the rotating magnetoconvection problem"*, Astronomische Nachrichten, 2020, 969-982.
4. Šoltis, T. and Brestenský, J., Rotating magnetoconvection with anisotropic diffusivities in the Earth's core. Phys. Earth Planet. Inter. 2010, 178, 27–38.
5. Filippi, E., Brestenský, J., and Šoltis, T., *"Effect of anisotropic diffusion on onset of rotating magnetoconvection in plane layer; stationary mode"*, Geophys. Astrophys. Fluid Dyn. 2019, 113, 80.
6. Fearn D. R., and Roberts P. H., *"The geodynamo, Chapter 4. In: Dormy E., Soward A.M. Mathematical Aspects of Natural Dynamos."* Taylor and Francis, 2007.
7. Gubbins, D., and Horrero-Bervera, E., *"Encyclopaedia of Geomagnetism and Paleomagnetism"*, springer, 2007.
8. Pesch, W. and Kramer, L., *"Nonlinear analysis of spatial structures in two-dimensional anisotropic pattern forming system"*, Z. Phys. B 63, 1986, 121.
9. Chandrasekhar, S., *"Hydrodynamics and hydromagnetic stability"*, Clarendon Press, Oxford, 1961.
10. Donald J. T., Roberts P. H., *"The effect of Anisotropic heat transport in the Earth's core on the Geodynamo"* Geophys. Astrophys. Fluid Dynam., 2004, 98, 5, 367–384.