Influence of spin slip condition on couple stress fluid flows : Exact solution

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Abstract

This paper concerns the solutions of classical flows namely Couette, Poiseuille & generalized Couette for incompressible, steady couple stress fluid flow with the help of slip conditions on velocity and couple stress. The influence for different parameters on volume flow rate and velocity profile of fluid is analyzed. It is obtained the limiting cases for each of the problem shows well agreement with previously published results of vanishing couple stress at boundary. The results show the presence of spin slip parameter decreases velocity and volume flow rate of the fluid.

Introduction and Objective

Couple stress fluid theory has vast applications in industrial and technological area, can be applied to model the flows of animal bloods, liquid crystals, lubrication and polymer thickened oils. This theory proposed by Vijay Kumar Stokes. According to this theory, the rotation vector defined as the half of curl of velocity vector and here, stress tensor is not symmetric [1, 2]. Many of flow problems of fluid flow problems have been studied with no-slip conditions. However, the situation might not always be realistic. Various researchers applied the slip conditions in study of different fluid flow problems. Navier [3] proposed a slip boundary condition, this ensures proportionality relation between tangential velocity of the fluid and shear rate at the solid surface. Neto et al. [4] discussed the Newtonian fluid flow with slip condition at solid surface experimentally. EI-Sapa et al.[5] examined movement of solid sphere in couple stress fluid flow with spin slip condition. Ashmawy [6] and Devakar et al. [7] applied slip boundary conditions at both upper and lower plates of micropolar fluid and couple stress fluid flow, respectively.

The main object of the paper is to investigate the effect of spin slip condition for three fundamental flow problems of couple stress fluid flow between parallel. This study has not examined earlier to the best of author's knowledge. The tangential and spin slip conditions are applied at both plates. Here, three cases have been analyzed: For the first case, we consider that both stationary plates are moving with various translatory constant velocities as well as fluid particles are rotating, also pressure gradient is zero in this case. In the second case, we assume the flow of couple stress fluid arises due to pressure gradient and fluid particles are rotating about their position. In the third case, fluid flow is caused by pressure gradient with the supposition of upper plate are moving with translatory velocity while lower plate is at rest and all the fluid particles are rotating.

Problem formulation

The physical model for unidirectional, steady and incompressible flow of couple stress fluid between parallel plates (situated at y = -h and y = h) is given below.

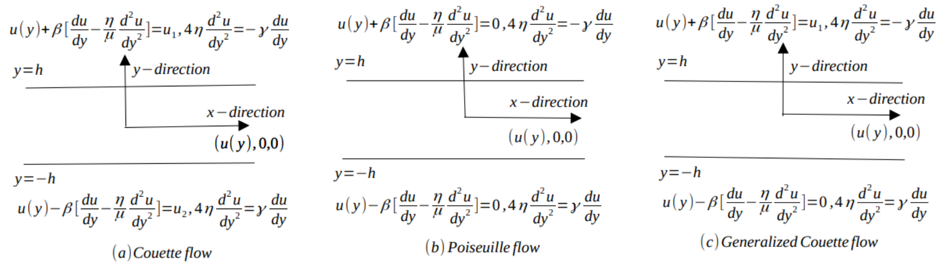

Figure 1 Couple stress fluid flow between parallel plates with slip and spin slip conditions

Governing equations for Couette, Poiseuille & generalized Couette flow are given by eqn. (1), (2) and (3) respectively

$$\eta\frac{d^4u}{dy^4}-\mu\frac{d^2u}{dy^2}=0 \ldots\ldots\ldots(1), \qquad \eta\frac{d^4u}{dy^4}-\mu\frac{d^2u}{dy^2}=-\frac{dp}{dx}\ldots\ldots\ldots(2), \qquad \eta\frac{d^4u}{dy^4}-\mu\frac{d^2u}{dy^2}=G\ldots\ldots\ldots(3)$$

Suppose velocity field can be taken as (u(y), 0, 0) & satisfy the continuity and momentum equations without body force and body couple.

Boundary conditions

Case 1: $u(-h)-\beta\left[\frac{du}{dy}-\frac{\eta}{\mu}\frac{d^3u}{dy^3}\right]\Big|_{y=-h}=u_2, u(h)+\beta\left[\frac{du}{dy}-\frac{\eta}{\mu}\frac{d^3u}{dy^3}\right]\Big|_{y=h}=u_1,\ 4\eta\frac{d^2u}{dy^2}\Big|_{y=-h}=\gamma\frac{du}{dy}\Big|_{y=-h},\ 4\eta\frac{d^2u}{dy^2}\Big|_{y=h}=-\gamma\frac{du}{dy}\Big|_{y=h}$

Case 2: $u(-h)-\beta\left[\frac{du}{dy}-\frac{\eta}{\mu}\frac{d^3u}{dy^3}\right]\Big|_{y=-h}=0, u(h)+\beta\left[\frac{du}{dy}-\frac{\eta}{\mu}\frac{d^3u}{dy^3}\right]\Big|_{y=h}=0,\ 4\eta\frac{d^2u}{dy^2}\Big|_{y=-h}=\gamma\frac{du}{dy}\Big|_{y=-h},\ 4\eta\frac{d^2u}{dy^2}\Big|_{y=h}=-\gamma\frac{du}{dy}\Big|_{y=h}$

Case 3: $u(-h)-\beta\left[\frac{du}{dy}-\frac{\eta}{\mu}\frac{d^3u}{dy^3}\right]\Big|_{y=-h}=0, u(h)+\beta\left[\frac{du}{dy}-\frac{\eta}{\mu}\frac{d^3u}{dy^3}\right]\Big|_{y=h}=u_1,\ 4\eta\frac{d^2u}{dy^2}\Big|_{y=-h}=\gamma\frac{du}{dy}\Big|_{y=-h},\ 4\eta\frac{d^2u}{dy^2}\Big|_{y=h}=-\gamma\frac{du}{dy}\Big|_{y=h}$

Where β and γ are tangential slip and spin slip coefficients, respectively.

Conclusions In case of Couette flow, fluid velocity increases with increasing effect in spin slip parameter near the lower plate however, this phenomena is reversed near the upper plate. For Poiseuille and generalized Couette flow, velocity of fluid decreases with increasing of spin slip parameter. In both cases, it is notice that the volume flow rate of fluid is decreasing function of spin slip parmeter. This type of trend have observed by [7] in their study.

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