Existence and Uniqueness of solution for stagnation point flow of a non-Newtonian Williamson fluid

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1. INTRODUCTION & OBJECTIVE

It is common knowledge that many industrial (such as paints and coatings) and physiological (such as blood and plasma) fluids exhibit complex flow behaviour that the classical Newtonian fluid model cannot adequately describe. In order to gain a better understanding of such fluids, several models (non-Newtonian) have been proposed over the years to take into account the unique characteristics of these fluids, including their viscoelastic properties, shear-thinning or shear-thickening behaviour, and time-dependent responses [1]. This paper focuses on the robust model put forward by Williamson to describe pseudoplastic fluids [2]. A large number of published works, for example, the investigation of thin film of pseudoplastic fluid moving over an inclined infinite solid surface [3], the peristaltic flow of chyme in the small intestine [4], blood flow through a tapered artery with a stenosis [5], and some boundary layer flows of Williamson fluid [6], to mention a few, demonstrate the adequacy of Williamson's model in describing many frequently observed industrial and physiological fluids like polymer solutions, paints, blood and plasma.

Different from the approaches used in the literature mentioned above, we aim to discuss the existence and uniqueness of a solution to the nonlinear equation describing the motion of Williamson fluid near the stagnation region of a solid surface. To the best of the authors' knowledge, only a limited number of articles are devoted to answering the question of the existence of a unique solution, see [7-9]. Under the boundary layer approximation and using suitable similarity transformation (as in [6]), we have considered the third-order nonlinear differential equation

$$F''' - F'^2 + FF'' + We\,\lambda\, F''F''' + 1 = 0, \tag{1}$$

with the boundary conditions

$$F(0) = 0,\ F'(0) = 0,\ F'(\infty) = 1, \tag{2}$$

where We, λ are the Weissenberg number and fluid parameter respectively. F is the transverse velocity.

RESULTS & HIGHLIGHTS OF IMPOINTANT POINTS

Following the works of [7], first, we establish the following Lemmas:

Lemma 1. P and Q are disjoint and open.

Lemma 2. P is non-empty.

Lemma 3. Q is non-empty.

Here, P and Q are subsets of $(0,\infty)$, defined by:

$$\mathrm{P} = \{\mathrm{a} > 0 : \exists\, \mathrm{s}_1 > 0 \text{ s.t. } \mathrm{F}''(\mathrm{s}_1;\mathrm{a}) = 0 \text{ and } 0 < \mathrm{F}'(\mathrm{s};\mathrm{a}) < 1 \text{ for } \mathrm{s} \in (0,\mathrm{s}_1]\},$$
$$\mathrm{Q} = \{\mathrm{a} > 0 : \exists\, \mathrm{s}_1 > 0 \text{ s.t. } \mathrm{F}'(\mathrm{s}_1;\mathrm{a}) = 1 \text{ and } 0 < \mathrm{F}''(\mathrm{s};\mathrm{a}) < \mathrm{a} \text{ for } \mathrm{s} \in (0,\mathrm{s}_1]\},$$

where $F(\mathrm{s};\mathrm{a})$ denote the solution of (1) with respect to the initial conditions $F(0) = 0,\ F'(0) = 0,$ $F''(0) = \mathrm{a}$ (free variable).

Then using the above Lemmas, we prove the existence of a unique solution to the non-linear equations (1)-(2).

Theorem 1. (Existence)
For any $We,\ \lambda \geq 0$ the equations (1)-(2) has a solution. Also the solution is monotone in nature.

Theorem 2. (Uniqueness)
For any $We,\ \lambda \geq 0,$ the solution is unique.

Further, numerical solution are obtained by using MATLAB BVP4C to support the properties of solution already proved.

Table 1 : Variation values of $F''(0)$ and $F(\infty)$ with We for $\lambda = 1$.

We	$F''(0)$	$F(\infty)$	$F(\infty)$ [6]
0.0	1.2325	4.3521	4.3541
0.2	1.1487	4.3228	4.3254
0.3	1.1158	4.3097	4.3123
0.5	1.0612	4.2858	4.2881

Fig 1: Variation of F' with We for fixed $\lambda = 1$.

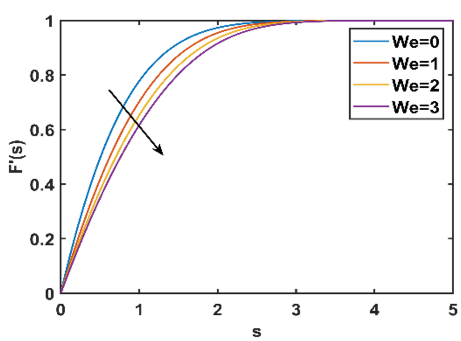
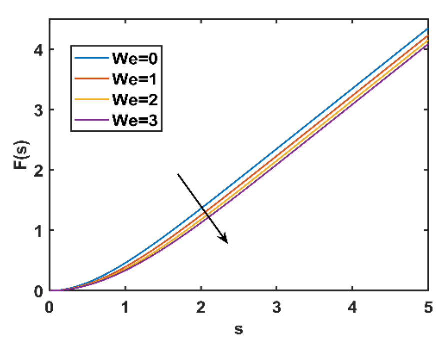


Fig 2: Variation of F with We for fixed $\lambda = 1$.



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