Finite Difference Approximation of Chemically Reactive Cross-Diffusion Effects on Double Diffusion Processes: Near Wall k-ε Turbulence Model

S.P. Suresha[1], G. Janardhana Reddy[1,*], H. P. Rani[2]

[1]*Laboratory on Computational Fluid Dynamics, Department of Mathematics, Central University of Karnataka, Kalaburagi-585367, India.*

[2]*Department of Mathematics, National Institute of Technology, Warangal-506004, India.*
*Corresponding author: gjr@cuk.ac.in

1. Abstract: This article emphasizes the cross-diffusion impacts on viscous dissipative and chemically reactive turbulent flow about a vertical cylinder embedded in a porous medium. The conservation laws of physics such as time mean continuity, momentum, temperature, and concentration equations considered in a 2D cylindrical coordinate system with additional transport equations, i.e. turbulence energy and dissipation rate of kinetic energy using a low Reynolds number k-ε model. PDEs which govern by turbulent flow are highly non-linear and coupled, which are amenable to solve any analytical methods. Therefore, the Crank-Nicolson scheme of FDM is employed which is stable unconditionally. The approximated solution in terms of simulated graphs studied with varying Re_t, Pr_t, Sc_t, Bu_t, and Gr_t. Also, these results good accordance with existing results.

2. Keywords: Turbulent flow, LRN k-ε model, FDM, Cross-diffusion effects, Natural convection.

3. Introduction: Turbulent flows are prevalent in various industrial applications, such as boundary layers forming on aircraft wings, combustion processes, and the flow of natural gas and oil in pipelines, as well as wakes behind ships, cars, submarines, and aircraft. Defining turbulent flow precisely is challenging due to its inherent complexity, which makes analytical methods difficult for its study. Instead, numerical methods are employed to investigate turbulent flows, as they offer more efficient ways to reduce both time and computational costs. The primary mathematical modeling approaches used in engineering for simulating turbulent flows include Direct Numerical Simulation (DNS), Large Eddy Simulation (LES), and Reynolds-Average Navier-Stokes (RANS) methods.

3. Results and Discussion: RANS procedure is used for constructing the governing equations. Algebraic equations obtained from FDM are solved using the Thomas algorithm. Simulated results are analyzed with varying turbulent Reynolds(Re_t) number is discussed below:

3.1. Average flow field: Fig. 1(a) illustrates the impact of Reynolds number (Re_t) on the unsteady time-averaged flow profile. The flow field starts from zero, reaches its maximum, and falls quickly, eventually becoming time-independent. Further, Fig. 1(b) shows the steady-state average flow field near a hot solid cylinder, where the velocity rises suddenly, reaches its maximum height, and gradually decreases to zero along the radial axis. Additionally, the time-averaged velocity field decreases in Fig. 1 as Re_t values increase.

3.2. Turbulence energy field: The variation of turbulence energy with Reynolds number (Re_t) is depicted in Fig. 2. In Fig. 2(a), the kinetic energy field near the cylinder momentarily merges due to heat transfer through conduction. In addition, Fig. 2(b) displays the time-independent kinetic energy profile, which increases near the hot cylinder due to rapid flow. Once the kinetic energy reaches its maximum, it decreases monotonically to zero along the radial axis. Furthermore, turbulence energy rises with an increase in Re_t since Re_t is directly proportional to kinetic energy.

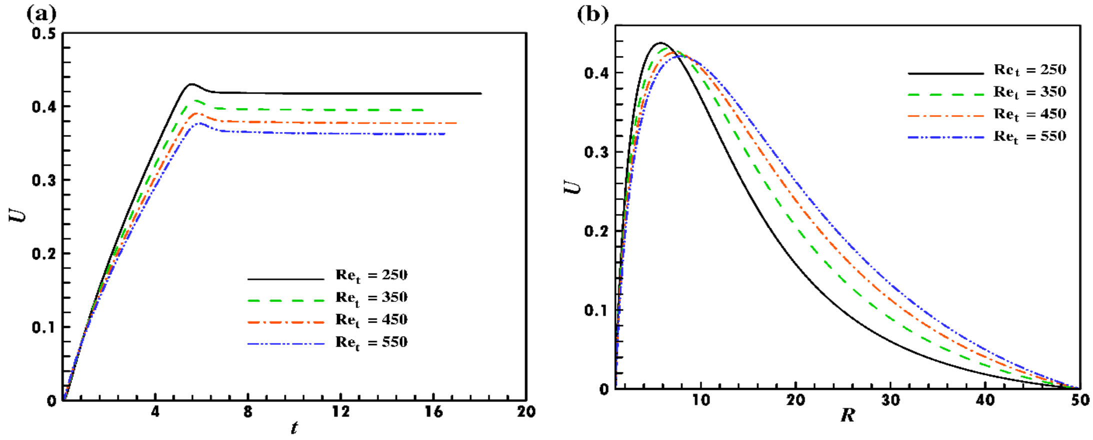

Figure-1: Time mean velocity field for different values of Re_t at (a) unsteady;(b) steady-state conditions with constant $Gr_t = 1.0$, $Sc_t = 0.7$, $Bu_t = 0.1$ & $Pr_t = 0.9$.

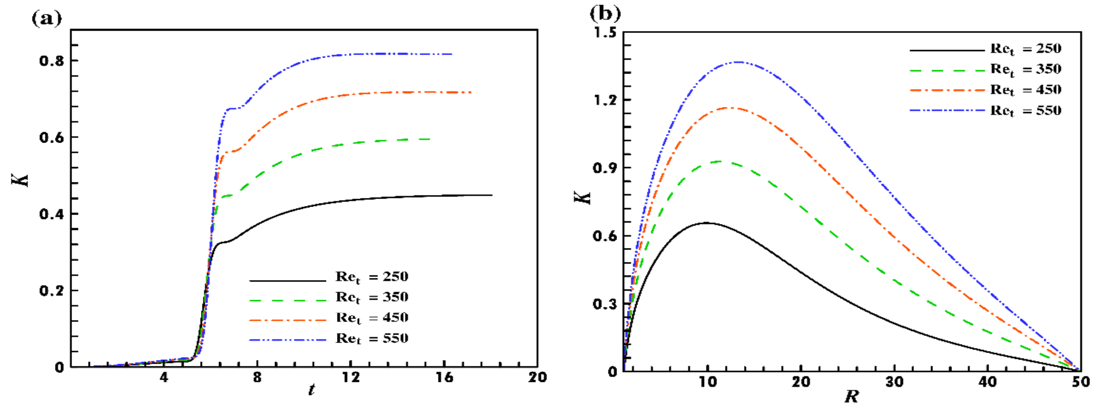

Figure-2: Time mean turbulence energy profile for different values of Re_t at (a) unsteady; (b) steady-state conditions with constant $Gr_t = 1.0$, $Sc_t = 0.7$, $Bu_t = 0.1$ & $Pr_t = 0.9$.

4. Reference:

1. Rani, H. P., Reddy, G. J., and Kim, C. N., Numerical analysis of couple stress fluid past an infinite vertical cylinder, ***Engineering applications of computational fluid mechanics,*** 5, 159-169 (2011).
2. Plumb, O. A., and Kennedy, L. A., Application of a k-ε turbulence model to natural convection from a vertical isothermal surface, ***Journal of heat transfer,*** 79-85 (1977).
3. Lin, S. J., and Churchill, S. W., Turbulent free convection from a vertical isothermal plate, ***Numerical heat transfer, Part B: Fundamentals,*** 1,129-145 (1978).
4. Fedorov, A. G., Viskanta, R., and Mohamad, A. A., Turbulent heat and mass transfer in an asymmetrically heated, vertical parallel-plate channel, ***International journal of heat and fluid flow,*** 18, 307-315 (1977).
5. Ouahabi, M. H., Ichenial, M. M., Hajjaji, A. E., and Benabdelouahab., Application of the turbulence models at low Reynolds number for horizontal-axis wind turbine design by using computational fluid dynamics, ***International journal of control and automation,*** 10, 179-190 (2017).
6. Eyub, C., Ali, Ates., and Şefik, B., Derivation of dimensionless governing equations for axisymmetric incompressible turbulent flow heat transfer based on standard k-ε model, ***Afyon kocatepe university journal of sciences and engineering,*** 20, 1096-1111(2020).