Linear and Energy Stability Analyses of Onset of Darcy-Bénard Convection Due to Combustion

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1. INTRODUCTION & OBJECTIVE

Combustion appears to be a simple and natural process, but reaction and fluid dynamics make it far more complicated to understand it. Frank-Kamenetskii (1938) has wonderfully described the theory of chemical reaction (combustion), showing how it is controlled solely by the energy equation during the initial stages of combustion. Frank-Kamenetskii (2015) proposed an energy equation for the later stages that essentially used the Arrhenius Law. The Frank-Kamenetskii parameter (FK) is typically introduced to characterize the explosion rate in steady-state problems, and beyond the critical value of FK , there is no steady-state solution. To maintain the steady-state temperature distribution in an exothermally reacting species, it is crucial to determine the range of applicability of FK in explosion issues (Adler, 1991). Dynamical analyses working closely with chemical reactions in a porous media have a long and illustrious history. Some of the classic research in this field are those by Gatica et al. (1987), Farr et al. (1991), and Malashetty et al. (1994). The current study aims to carry out linear and nonlinear analyses of Darcy-Bénard convection (DBC) in a Newtonian fluid going through combustion such that the energy equation follows the Frank-Kamenetskii formulation. The problem is subjected to analyze how the Frank number (FK) impacts the accuracy of the two analyses' predictions about the beginning of DBC.

Mathematical Formulation: The governing equations and boundary conditions taken into account are expressed in dimensionless form as follows:

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

$$\mathbf{q} = -\nabla P + \left(\frac{K\, d\, \rho_0}{\mu\, \alpha_m} g - Ra_D^{FK}\, \theta \right) \hat{k}, \tag{2}$$

$$\frac{\partial \theta}{\partial t} + \left(\mathbf{q} \cdot \nabla \right) \theta = \nabla^2 \theta + FK \exp\left(\theta \right), \tag{3}$$

$$\mathbf{q} = \theta = 0 \qquad at \qquad z = 0,1. \tag{4}$$

Methodology: The periodic boundary condition for longitudinal rolls is the one to be taken into account when z is involved. The critical Frank-Kamenetskii-Darcy-Rayleigh number $\left(Ra_{Dc}^{FK} \right)$ is obtained via normal mode analysis. The following BEVP is obtained from the analysis:

$$\left(4D^2 - a^2 \right) W + a^2 \sqrt{Ra_D^{FK}}\, \Theta = 0, \tag{5}$$

$$\sqrt{Ra_D^{FK}}\, D\theta_b W + \left(4D^2 - a^2 \right) \Theta + FK \exp\left(\theta_b \right) \Theta = 0, \tag{6}$$

$$W = \Theta = 0 \qquad at \qquad z = -1,1. \tag{7}$$

2. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

The nonlinear stability analysis is taken into account in this paper to determine whether the combustion in the fluid has any significant nonlinear effects that contribute to subcritical instability. We use the generalized energy approach to undertake a nonlinear analysis to better comprehend this situation. The energy equation is constructed in terms of kinetic and thermal energies. Figure 1 illustrates subcritical motions that emerge from the combustion problem's non-linear fundamental temperature gradient caused by heat generation from exothermic chemical reactions. The results of the linear and nonlinear studies overlap when $FK = 0.1$, while larger values of FK cause a noticeable shift. Small values of FK indicate a less efficient chemical reaction process and a predominance of conduction, which prevents the instability caused by nonlinear terms from manifesting.

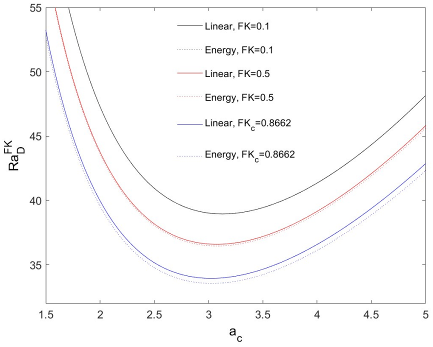

Figure 1 Neutral stationary stability curve for different values of FK .

Table 1 illustrates how the critical Frank-Kamenetskii-Darcy-Rayleigh number $\left(Ra_D^{FK}\right)_c$ and wave number $(a)_c$ vary for various FK values in the cases of linear and nonlinear stability. Table 1 demonstrates that the system is more unstable when there is combustion present than when it is not. Due to the development of extremely high temperatures, the system may explode after a certain value of FK . This fact is supported by the observation that wavenumber values decrease as combustion progresses.

Table 1: Variation in critical Rayleigh $\left(Ra_D^{FK}\right)_c$ number and wavenumber $\left(a_c\right)$ for different values of FK .

			Linear Analysis		Nonlinear Analysis		
FK	c_1	c_2	a_{cL}	$\left(Ra_D^{FK}\right)_{cL}$	λ	a_{cE}	$\left(Ra_D^{FK}\right)_{cE}$
0.1	1.35113	0.12799	3.12831	38.96213	1.00	3.12840	38.95670
0.3	2.10245	0.35726	3.10190	37.84471	0.98	3.10270	37.79735
0.5	2.93048	0.57593	3.07533	36.60381	0.96	3.07748	36.47166
0.7	3.85298	0.80034	3.04788	35.22456	0.94	3.05197	34.96658
0.8662	4.70915	0.99996	3.02358	33.95867	0.91	3.02916	33.56763

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