Non-linear oscillation of a neo-Hookean material hyperelastic incompressible tube using the Lie-point symmetry

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1. Abstract

A transverse isotropic hyperelastic material constitutive equation is expressed in covariant form for any aspect of the anisotropic director. For radial oscillations in three transverse isotropic cylindrical tubes of generalized neo-Hookean materials, a non-linear differential equations are developed. An analysis of Lie point symmetry is done. For Lie-point symmetries to obtain, certain requirements on the strain-energy function and the net applied surface pressure must be determined. The differential equation is simplified to differential equation for these transverse isotropic tubes. The Ermakov-Pinney equation describes radial oscillation in a transverse longitudinal isotropic tube.

Keywords: Transverse isotropic hyperelastic materials; Anisotropic director; neo-Hookean materials; Ermakov-Pinney equation; Lie-point analysis

2. Introduction

Solid mechanics examines how deformable bodies respond mechanically to various kinds of external influences. For the past several decades, new research has been steadily adding to and developing the theory of elasticity for anisotropic bodies. Materials that have a single response in one direction and a different response in the other two are referred to as transversely isotropic materials. These materials are described using five distinct elastic constants. Different constitutive equations are utilized for each condition in the classical treatment, which is based on a hypothesis that simplifies the issue. In order to understand the theory of infinitesimal strain to be applicable, minor deformations are first assumed. Second, assuming the material is incompressible simplifies the constitutive equation for the material.
Seth (1962–1964) created the elastic deformation transition theory. It has been argued by numerous authors in the literature that the change from one state to another is an asymptotic process. According to Seth, the differential system ought to reach some criticality at the transition. The asymptotic solutions at the transition points, once the critical point has been identified, provide the solution corresponding to the transition state that is ignored in the classical theory. Numerous issues involving cylinders, disks, shells, etc. are addressed by this theory under various loading functions.

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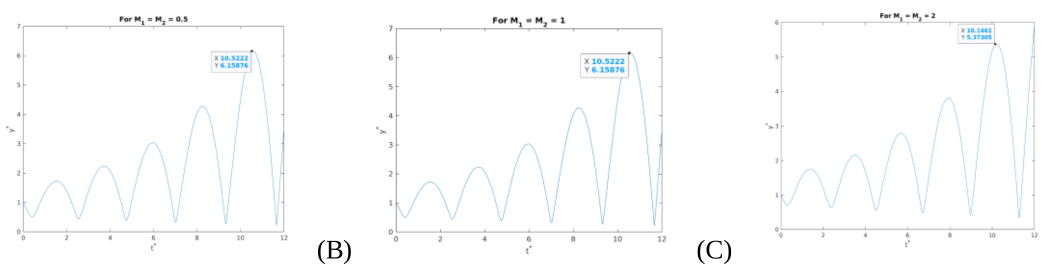
3. Results & Discussion

The differential equations governing non-linear radial oscillations, in a thick-walled cylindrical tube. It is shown in terms of the strain-energy function W. For thin-walled tubes, the approximation resembles that of isotropic tubes and holds for tangential, longitudinal, and radial transverse isotropic tubes as well. Three second-order differential equations for radial oscillation in thin-walled tubes are derived when a generalized neo-Hookean strain-energy function is used. The Ermakov-Pinney equation is obtained for a longitudinal transversely isotropic tube. The ordinary differential equations explains radial oscillations in radial and tangential transversely isotropic thin-walled tubes. When the total applied surface pressure varies with time these two transversely isotropic tubes exhibits one Lie point symmetry. This symmetry holds for specific net applied surface pressures and certain forms of generalized neo-Hookean strain energy functions. The effects of different defined parameters on these differential equations are investigated.

The Lie point symmetry generator allows the transformation of the differential equation into a second-order Abel equation. Figures illustrate the significant reduction in the transformed inner radius as time progresses, showing the influence of defined parameters. This research contributes to a better understanding of neo-Hookean hyperelastic materials.



(A) (B) (C)

Figure.1. Transformed inner radius y* plotted for transformed time t* for (A) $M_1 = M_2 = 0.5$ (B) $M_1 = M_2 = 1$ (C) $M_1 = M_2 = 2$ subjected to initial condition y*(0) = 1 (y(0) = 1) and $\frac{dy*}{dt*}(0) = -\frac{4}{3}$ ($\frac{dy*}{dt*}(0) = -1$) for radial oscillation in a transverse isotropic hyperelastic tube.

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