Poiseuille flow of some non – Newtonian fluids under the influence of inclined magnetic field

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1. INTRODUCTION

Non – Newtonian fluid flow can be found in many industries and manufacturing plants [1]. These fluids are basically nonlinear. The constitutive equations relating to such fluids are intrinsically more sophisticated than conventional Newtonian fluids. Examples of non – Newtonian fluids include blood, shampoo, etc. The flow of non – Newtonian fluids in a channel under the influence of a magnetic field is an important problem and has been studied by many researchers. The magnetic field can alter the flow structure, velocity profile, pressure distribution and heat transfer characteristics of the fluid [2 – 5].

The last decade has seen a surge in the development of neural network and its various applications. A neural network developed with a specific view of solving the boundary value problems is the Physics Informed Neural Network (PINN). The PINN has been observed to be very accurate and serves the purpose of solving various types of boundary value problems arising in science and engineering fields [6 – 8].

In this paper, an attempt is made to solve Poiseuille flow of some non – Newtonian fluids under the influence of an inclined magnetic field using PINN. To check the accuracy and validity of the PINN, the governing equations are also solved using the spectral collocation method.

2. MATHEMATICAL FORMULATION

We consider the flow of three types of non – Newtonian fluid in a channel filled with a saturated porous medium under the influence of an inclined magnetic field. The fluids considered are Casson, Jeffrey and Williamson fluids, which are all non – Newtonian in nature. The flows are assumed to be laminar, fully developed and Boussinesq approximation is assumed. The walls of the channel are assumed to be at different temperatures. Thus, the non – dimensional momentum equations of the flow are as follows:

Casson: $\left(1+\frac{1}{\beta}\right)\frac{d^2U}{dY^2}-\frac{U}{Da}-Ha^2U\sin^2\theta+A+\frac{Gr}{Re}T=0$

Jeffrey: $\left(\frac{1}{1+\lambda}\right)\frac{d^2U}{dY^2}-\frac{U}{Da}-Ha^2U\sin^2\theta+A+\frac{Gr}{Re}T=0$

Williamson: $\frac{d^2U}{dY^2}+2We\frac{dU}{dY}\frac{d^2U}{dY^2}-\frac{U}{Da}-Ha^2U\sin^2\theta+A+\frac{Gr}{Re}T=0$

The energy equation is assumed as: $\frac{d^2T}{dY^2}=0$

Boundary conditions: $U(0)=0\,;U(1)=0\,;T(0)=r_T\,;T(1)=1$

where $Da = \frac{K}{L^2}$ is Darcy number; $Ha = B_0 L \sqrt{\frac{\sigma}{\mu}}$ is Hartmann number; $A = -\frac{L^2}{\mu U_0}\frac{dp}{dx}$ is pressure gradient; $Gr = \frac{g\beta(t_2 - t_0)L^2}{\nu^2}$ is Grashoff number; $Re = \frac{U_0 L}{\nu}$ is Reynold's number; $We = \frac{\Gamma U_0}{\mu L}$ is Weissenberg number.

3. METHODOLOGY

The equations are solved using a Physics Informed Neural Network (PINN) method. PINNs are a type of machine learning approach that can be used to find the solution of differential equations by including all the physics into the loss function and building a neural network that approximates the solution. The fundamental idea of PINN is that the neural network approximates the solution of a given differential equation and satisfies any constraint such that the loss is minimized. This approach has been shown to be very effective in solving various types of differential equations like fractional equations, integral – differential equations and stochastic PDEs. To affirm the correctness and accuracy of the PINN method, we also solve the flow equations using spectral collocation method.

4. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

The emphasis of the current work is to solve the non – Newtonian fluid flows using the PINN method. The solutions are then cross checked using the spectral collocation method. The effect of flow parameters like Darcy number, Hartman number, magnetic field angle on the flow velocity is discussed with the supplement of visual representations.

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