Effect of Axial Load on Static and Eigenvalue Characteristics of MEMS with Compliant Support

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1. INTRODUCTION & OBJECTIVE

Electrostatically-driven microelectromechanical systems (MEMS) devices are found in many appplications such as ultra sensitive sensors, signal processing, nano-switches, etc. These devices are manufactured using microfabrication processes and which, induces internal stresses in the MEMS resonators. These MEMS devices are mathematicaly modeled as slender beams. It has been seen that the internal stresses arising from microfabrication processes are often encountered in doubly clamped microbeams. In this work, the microbeam is modeled using Euler-Bernoulli beam theory with geometric nonlinearity (due to beam stretching) and forcing nonlinearity (due to electrostatic forcing). In addition, the supports are modeled with non ideal boundary conditions (compliant support). These compliant supports are modeled as transverse and rotational springs in this work. The governing equations are solved using a Galerkin based reduced order modeling (ROM) technique. The objective of this work is to study the effect of internal stresses and the compliant support on the natural frequency and mode shapes of the microbeam. Further, the static and dynamic characteristics of these microbeams will be studied in context of internal stresses and compliant supports.

2. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

The configuration of a microbeam (movable electrode) with compliant support is shown in Fig.1. The governing differential equation and boundary conditions are given in Eq.1.

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = \left[\frac{AE}{2L}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx + N\right]\frac{\partial^2 w}{\partial x^2} + F_e$$

$$at\ x = 0 \quad EI\frac{\partial^2 w}{\partial x^2} - K_R\frac{\partial w}{\partial x} = 0 \qquad at\ x = L \quad EI\frac{\partial^2 w}{\partial x^2} + K_R\frac{\partial w}{\partial x} = 0 \qquad (1)$$

$$EI\frac{\partial^3 w}{\partial x^3} - N\frac{\partial^2 w}{\partial x^2} + K_T w = 0 \qquad EI\frac{\partial^3 w}{\partial x^3} - N\frac{\partial^2 w}{\partial x^2} - K_T w = 0$$

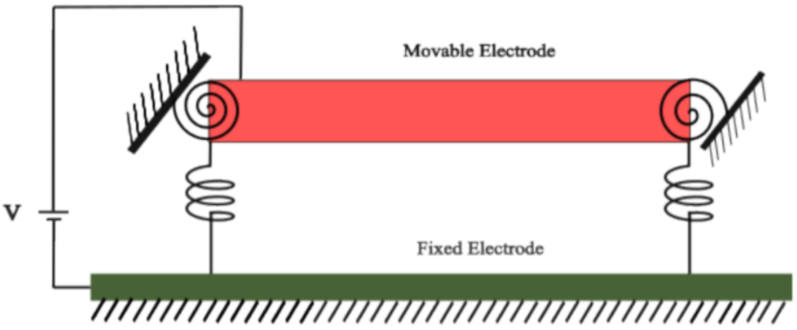

Figure 1: MEMS Resonator with complinat support

Where, $\boldsymbol{w}$, $\boldsymbol{E}$, $\boldsymbol{I}$, $\boldsymbol{L}$, $\boldsymbol{\rho}$, $\boldsymbol{A}$, $\boldsymbol{K_R}$, $\boldsymbol{K_T}$, $\boldsymbol{N}$, $\boldsymbol{F_e}$ are the transverse deformation, Young's modulus, moment of inertia, length, material density, Area of cross-section, rotational stiffness, transevers stiffness, internal axial stress and electrostatic force in the microbeam, respectively. Governing equation is non-dimensionalised using scaling parameters and ROM is developed by considering first few mode shapes. System of nonlinear algebraic equations is solved in MATLAB numerically to to get static deflection. Linear eigenvalue problem is solved using *eig* command in MATLAB to get linearized eigenvalues for applied electrostatic force. Effect of compliant support and internal axial stress on natural frequency, mode shapes and pull-in characteristics is shown in Fig.2. $\boldsymbol{\alpha_R}$, $\boldsymbol{\alpha_T}$ and $\boldsymbol{N_{b1}}$ are the non-dimensional rotational stiffness, transvese stiffness and first euler buckling load respectively. Rotational stiffness decreases the value of natural frequency and changes the slope at fixed end. Pull-in characteristics for MEMS resonator with compliant support is analysed for different values internal stress. Increasing axial stress from compressive to tensile increases pull-in voltage and natural frequency at deflected position $(\boldsymbol{\omega_{d1}})$ but decreases pull-in displacement. For compressive internal axial stress we observed the frequency tunning property.

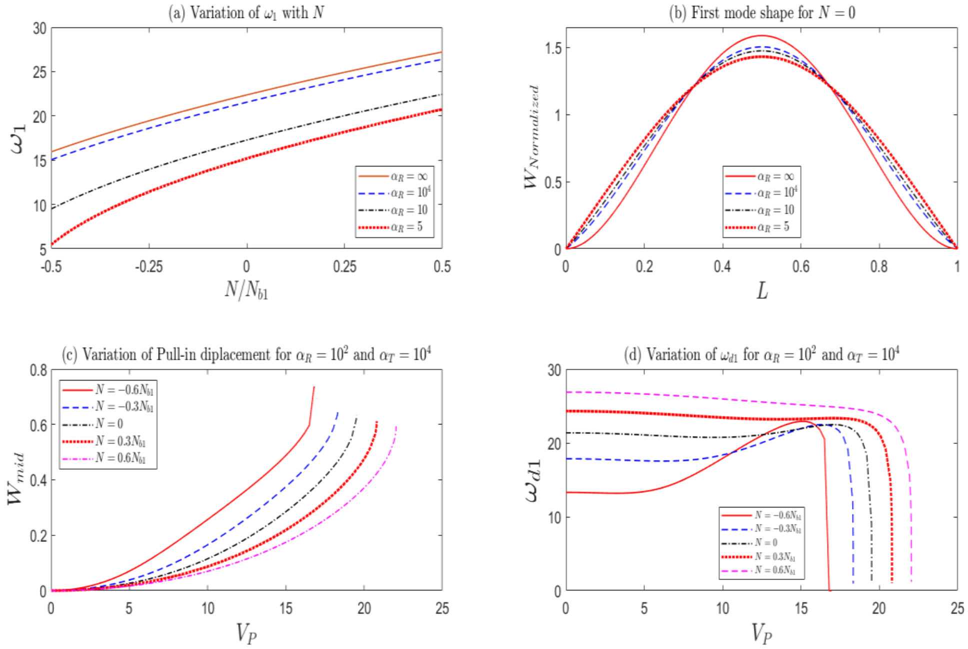

Figure 2: Effect of compliant support and internal stress on the natural frequency, mode shapes and pull-in.

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