Water wave scattering by a porous disk submerged in a two-layer fluid

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1. INTRODUCTION

Over the past few decades, a lot of attention has been paid to the problem of very large floating structures (VLFS) as different offshore structures such as floating airports, oil storage, wind and solar power plants, etc. (Das and Sahoo 2017). A review on recent advances regarding the importance of VLFSs and the extensive research activities can be found in Lamas-Pardo et al. (2017). To reduce the wave loads on the flexible structures, often structural porosity is introduced. It helps to dissipate a major part of the incoming wave energy. Recently, Meylan et al. (2017) used eigenfunction expansion method for the hydroelastic analysis of a circular flexible porous plate. Further, a coupled boundary element and finite element method is used to study the hydroelastic behaviour of a floating plate having arbitrary shape.

Although significant works have been devoted to study the wave load on a floating platform, limited emphasis have been focused on understanding the effect of porosity on the hydrodynamic forces acting on the submerged disk in a two-layer fluid. Hence, in this work we will analyze the effect of porosity on the vertical wave force acting on the porous disk using eigenfunction expansion method.

MATHEMATICAL FORMULATION

We consider here a floating elastic plate of uniform thickness and negligible draft. The plate is assumed to be circular with radius a. The upper fluid layer of constant density ρ_1 occupies the region $-\infty < x, y < \infty, -h < z < 0$, with $z = 0$ is the mean free surface. The lower fluid layer of constant density ρ_2 occupies the region $-\infty < x, y < \infty, -H < z < -h$, with $z = -h$ is the mean interface. The porous disk is assumed to be placed in the upper fluid layer at $z = -d < 0$. Let ϕ_1 and ϕ_2 are the velocity potentials in the upper and lower fluid layers respectively. Assuming linear water wave theory, the governing equations and boundary conditions are given by

$$\nabla\phi_j = 0,$$

$$\partial_z\phi_j = 0, \; z = -H,$$

$$\partial_z\phi_j = K\phi_j, \;\; z = 0,$$

$$\partial_z\phi_1 = \partial_z\phi_2, \qquad z = -h,$$

$$\rho_1(\partial_z\phi_1 - K\phi_1) = \rho_2(\partial_z\phi_2 - K\phi_2), \;\; z = -h,$$

where $K = \frac{\omega^2}{g}$ with g is the acceleration due to gravity and ω is the frequency. The velocity and pressure matching conditions on the disk are

$$\partial_z \phi_2^+ = \partial_z \phi_2^- = i\sigma(\phi_2^+ - \phi_2^-), \qquad z = -d,$$

Applying the separation of variables technique to the velocity potentials, the vertical eigenfunctions can be written as

$$Z_1(\mu_n z) = \frac{\cosh \mu_n(z+H)}{\cosh \mu_n H}, \ \ n > 0, \ \text{and} \qquad Z_2(k_n z) = \frac{\cosh k_n(z+H)}{\cosh k_n H}, \ \ n > 0,$$

where, Z_1 and Z_2, respectively, are the vertical eigenfunctions in the plate and water region. k_n satisfies the dispersion equation in the water region where k_0 and 1 corresponds to the positive real roots and $k_n, n > 2$ denote the imaginary roots corresponding to the evanescent modes. The porous nature of the disk gives rise to complex wavenumbers, which are in close proximity to the roots of the corresponding equation for non-porous plate.

METHOD OF SOLUTION

We derive the solution here by using eigenfunction expansion method in cylindrical coordinates.

For computational purpose the steps are as follows:

1. Solve the truncated system of equations for a given set of parameters.
2. Evaluate the vertical force acting on the plate using those unknowns.
3. Analyze the effect of different plate parameters on wave force by using numerical computations.

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