Nonlinear wave propagation through a nonideal dusty gas with mixed nonlinearity

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1. ABSTRACT

We examine a quasilinear system of hyperbolic differential equations governing the one-dimensional, unsteady flow of a dusty gas; the equation of state of the medium is that of a van der Waals gas and the fundamental derivative Γ is of order $O(\epsilon)$ and changes sign about the base state Γ=0. Using the method of multiple scales the system is reduced to an inviscid Burger's equation having quadratic and cubic nonlinearities and a source term. The effects of dust particles and the van der Waals excluded volume on the evolution of the pulse are studied analytically and numerically using the WENO scheme.
Keywords: hyperbolic system, van der Waals gas, mixed nonlinearity, dust particles, shock.

2. INTRODUCTION & OBJECTIVE

In many astrophysical problems, we encounter high speed flow in a mixture of gas and small solid particles resulting in the formation of shock. This might be the result of collision of a coma with a planet. Pai, Menon and Fan[1] have extended the problem of a point explosion in a gas to a two-phase flow of a mixture of a gas and small solid particles and examined the effects of dust particles on the resulting strong shock wave. Jena and Sharma[2] have established the entire class of self-similar solutions to the problem of shock wave propagation through a dusty gas. Chadhha and Jena[3] have studied various aspects of one dimensional unsteady plane flow of an inviscid nonideal gas with dust particles.
However, the fundamental derivative Γ of the medium has been assumed to be positive for all these investigations. The behaviour of a real fluid differs from that of an ideal fluid significantly particularly with respect to the shock wave phenomenon [4]. In the present study we have considered wave propagation through a dusty gas exhibiting mixed nonlinearity. It is assumed that the fundamental derivative Γ associated with the medium is of order $O(\epsilon)$, $0<\epsilon<<1$ and changes sign about the base state Γ=0, where $\Gamma(\rho,s)=\frac{1}{\rho}\frac{\partial}{\partial\rho}(a\rho),\ a=\sqrt{\frac{\partial p}{\partial\rho}}\Big|_s$ is the local speed of sound, $p=p(\rho,s)$ is the pressure, ρ the density and s, the entropy of the fluid.
In perfect gases and longitudinal waves in solids Γ is strictly positive and only compression shocks are formed in such fluids; two of the earliest studies pertaining to the existence of expansion shocks in gases are due to Bethe [5] and Zeldovich [6] who showed that for sufficiently large values of specific heats van der Waals gases admit expansion shocks. Subsequently Thompson and Lambrakis [7] established the presence of single-phase fluids with $\Gamma<0$.
The principles of the method of Geometrical optics have been successfully extended to wave motion by Choquet Bruhat[8], Hunter and Keller[9], Majda and Rosales[10], Cramer and Sen[11] , Kluwick and Cox[12] and Shukla and Sharma[13]. This technique involves the introduction of slow and fast variables and phase functions; the precise scaling of the fast variables in comparison to the slow ones depends on the problem under study and varies from one problem to another. We have employed the analytic apparatus of Kluwick and Cox[14]

wherein the signal is assumed to propagate with a perturbed strength $O(\epsilon)$ and the disturbance appears after a time $O\left(\frac{1}{\epsilon^2}\right)$.

Using the method of multiple scales we examine the propagation of small amplitude high frequency waves through a van der Waals gas suspended with dust particles and obtain a transport equation which determines the final evolution of the signal. The effects of dust particles and the van der Waals excluded volume on wave propagation are studied analytically and numerically.

3. RESULTS & HIGHLIGHTS OF IMPOINTANT POINTS

The nonlinear geometric ray theory yields an evolution equation which is a Burger-like equation with quadratic and cubic nonlinearities and a source term; the quadratic nonlinearity can be identified with Lax's genuine nonlinearity while the cubic nonlinearity is a measure of the material nonlinearity of the medium. The source term is a function of the wavefront curvature and the frozen sound speed that delay the shock formation. The sound speed which is also present in the flux function varies with the parameters associated with dust particles and the van der Waals excluded volume. Numerical experiments using the WENO scheme demonstrate the effect of these parameters on the wave evolution.

4. REFERENCES

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