# Thermocapillary migration of a compound drop inside a spherical cavity

Chennuri Dhanya, Jai Prakash
Department of Mathematics,
École Centrale School of Engineering, Mahindra University, Hyderabad -500043, India

## 1. INTRODUCTION AND OBJECTIVE

In this paper, our purpose is to obtain numerical results for the thermocapillary migration of a compound drop inside a spherical cavity in which the uniform temperature gradient is imposed along the $z$-axis. Furthermore, the study takes into account the impact of the thermal conductivity of the boundary phase in the axisymmetric thermocapillary motion of the compound drop within the concentric spherical cavity, a linear temperature field $T_\infty(z)$ with uniform gradient $E_\infty e_z = \nabla T_\infty$ is imposed in the cavity surroundings far away from the compound drop and $E$ is taken to be positive. Before determining the thermocapillary migration, it is necessary to obtain temperature and velocity distributions which are axisymmetric in all the phases, and the Marangoni number is assumed small. The flow fields in each drop phase are governed by Stokes equations, whereas the thermal problem is harmonic function. The hydrodynamic problem and thermal problem are coupled through specific boundary conditions. A complete general solution of Stokes equation is used to derive closed form expressions for the velocity and pressure. The hydrodynamic force experienced by the compound drop is obtained by integrating the viscous stress over its surface.

This primary goal is to assess the mobility of a compound drop within a spherical cavity under the influence of an imposed temperature gradient. The thermocapillary motion of the compound drop is affected by the presence of a rigid spherical boundary, making the migration behavior more pronounced due to the axial symmetry. By employing analytical methods, the study derives a closed-form expression for the thermocapillary mobility of the confined compound drop. To determine the unknown velocity of the compound drop, the force-free condition is applied, ensuring that it translates without experiencing any net external forces.

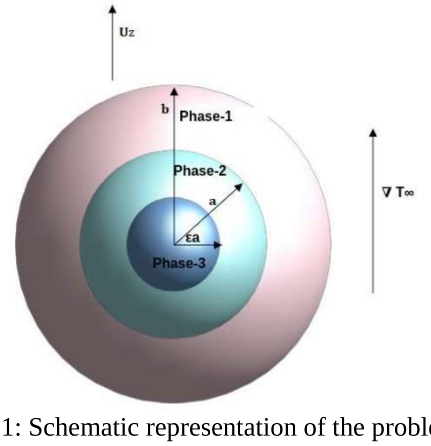
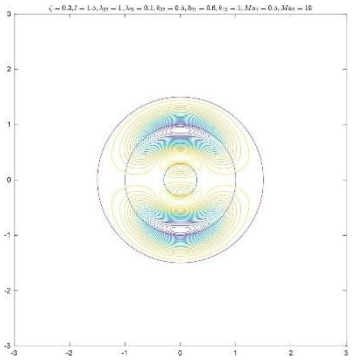


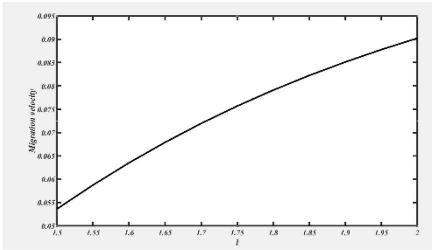

Figure 1: Schematic representation of the problem

## 2. RESULTS AND HIGHLIGHTS OF IMPORTANT POINTS

The normalized thermocapillary migration velocity with respect to the thermocapillary migration velocity of a compound drop in an unbounded medium is presented for various values of the parameters such as viscosity ratio, Marangoni numbers etc. Figure 2 (B) presents the normalized migration velocity which is nothing but the rate of movement of compound drops due to thermocapillary effects, to be an increasing function of the ratio of the compound drop's radius to the cavity radius. It can be further observed from Fig. 3 (A) that the normalized migration velocity is an increasing function of viscosity ratio $\lambda_{21}$ and will become infinite in the limit $\lambda_{21} = 1$ for any value of other parameters. Interestingly, when the value of $\lambda_{21}$ is little above unity, $U/U_0$ first decreases and reaches a minimum at some finite value of $\lambda_{21}$ and afterwards it starts increasing.

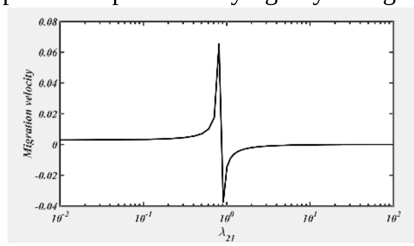


(A)

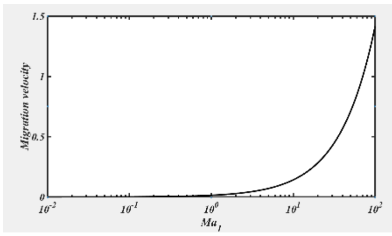



(B)

Fig 2: (A). Streamlines of a compound drop located concentrically in a spherical cavity $l = b/a = 1.5$, $\lambda_{32} = \mu_3 / \mu_2 = 1$, $\lambda_{21} = \mu_2/\mu_1 = 0.1$, $K_{32} = k_3 / k_2 = 0.5$, $K_{21} = k_2/ k_1 = 0.6$, $K_{10} = 1$, $Ma_1 = 0.5$, $Ma_2 = 10$, (B). Migration velocity of compound drops with varying $l$ by fixing $\lambda_{32} = 1$, $\lambda_{21} = 0.5$, $K_{32} = 0.5$, $k_{21} = 0.5$, $K_{10} = 0.5$, $Ma_1 = 0.5$, $Ma_2 = 1$.




(A)




(B)

Fig 3: (A). Migration velocity of compound drop with varying $\lambda_{21}$ by fixing $l = 1.5$, $\lambda_{32} = 1$, $k_{21} = 0.5$, $K_{32} = 0.5$, $K_{10} = 0.5$, $Ma_1 = 0.5$, $Ma_2 = 1$ (B). Migration velocity of compound drop with varying $Ma_1$ by fixing $l = 1.5$, $\lambda_{32} = 1$, $\lambda_{21} = 0.5$, $K_{32} = 0.5$, $K_{10} = 0.5$, $k_{21} = 0.5$, $Ma_2 = 1$.

## References

1. Han C Chiu and Huan J Keh. "Thermocapillary migration of a fluid sphere in a circular tube." In: *American Journal of Heat and Mass Transfer* 3.1 (2016), pp. 15–36.
2. Tai C Lee and Huan J Keh. "Axisymmetric thermocapillary migration of a fluid sphere in a spherical cavity." In: *International Journal of Heat and Mass Transfer* 62 (2013), pp. 772–781.
3. Tai C Lee and Huan J Keh. "Thermocapillary motion of a spherical drop in a spherical cavity." In: *CMES* 93 (2013), pp. 317–333.