A hybrid phase-field framework to model crack propagation in poroelastic media

Salman Khan[1*], Chandrasekhar Annavarapu[1] and Antonio Rodríguez-Ferran [2]

[1]Department of Civil Engineering, Indian Institute of Technology Madras, Chennai, TN, 600036, India
[2] Laboratori de Càlcul Numèric (LaCàN), Universitat Politécnica de Catalunya, Barcelona, 08034, Spain

ABSTRACT

We propose a hybrid phase-field formulation to simulate crack propagation in poroelastic media. The proposed approach is a three-field formulation (displacements, pressure, and damage as the field variables) with three governing field equations (linear momentum balance, mass balance, and damage evolution). The constitutive response of the medium is modeled as per Biot's theory of linear poroelasticity. The proposed approach extends the hybrid phase-field method of Ambati et al. (Computational Mechanics, 2015; 55(2): 383–405) to poroelastic materials. We demonstrate that, for such materials, a standard implementation of the phase-field method may yield either (a) negative damage, (b) damage larger than 1, and (c) reversible damage. This non-physical behavior is exhibited because the history field variable used in the standard phase-field method was designed for purely mechanical problems and did not include the additional contributions from the product of pressure and volumetric strain. To remedy this issue, we introduce a new history field variable (a pressure history field) to ensure the following constraints on damage: (a) damage is bounded between 0 and 1, (b) damage exhibits monotonic growth, and (c) when pressure is zero, the contributions from the pressure history field vanish. We compare the results obtained using the proposed formulation with the hybrid formulation without the proposed modifications for several benchmark problems. The results demonstrate the advantages of the proposed model for modeling crack propagation in poroelastic materials.

1. INTRODUCTION

Recently, the hybrid phase-field model[4] has become increasingly popular to model brittle fracture propagation in elastic materials. The hybrid phase-field model contains attractive features from the more advanced phase-field models, such as tension-compression splitting and damage irreversibility, while keeping the sub-problems of equilibrium and damage growth linear when solved in a staggered manner. However, the method is yet to be applied to model fracture propagation in poroelastic media. The modeling and simulation of fracture propagation in poroelastic media are important for various engineering fields, including geotechnical engineering, petroleum engineering[1], biomedical engineering[2], and glaciology[3]. As such, this study focuses on the extension and subsequent application of the hybrid phase-field model to study fracture propagation in poroelastic media. Firstly, we demonstrate that a direct application of the hybrid model to coupled poroelastic problems violates the constraints on damage lower and upper bounds and irreversibility. Subsequently, we propose a modification of the history field variable to incorporate the effect of pressure. The proposed modification ensures that damage growth is irreversible, and that damage always remains bound between 0 and 1. Additionally, we also show that when pressure is zero, the proposed history field variable is identical to the one used in the original hybrid formulation for purely mechanical problems.

2. METHODOLOGY

We employ classical Biot's poroelasticity[6] to describe the interactions between fluid flow and solid deformation. The two main governing equations (i.e., the momentum balance and mass balance) for the poroelasticity problem are summarised as follows:

$$\nabla \cdot \boldsymbol{\sigma} = 0 \quad \text{with} \quad \boldsymbol{\sigma} = \boldsymbol{\sigma}' - \alpha p \boldsymbol{I} = 0 \tag{1}$$

$$\frac{1}{M}\frac{\partial p}{\partial t} + \alpha \nabla \cdot v_s + \nabla \cdot (\kappa \nabla p) = 0 \tag{2}$$

where p is the pore-pressure in the domain, M and α are Biot's modulus and coefficient, v_s is solid grain velocity, κ is hydraulic conductivity, $\boldsymbol{\sigma}'$ is effective stress.

The fracture evolution in the porous medium is governed by the following diffusion equation:

$$-\ell^2 \nabla^2 d + d = \frac{2\ell}{G_c}(1-d)\mathcal{H}_m^+ - \frac{\ell}{G_c}\mathcal{H}_p \tag{3}$$

where the term ℓ is the phase-field length scale, G_c is critical energy release rate, while $\mathcal{H}_m^+ = \max(\psi_0^+(\boldsymbol{\varepsilon}, d))$ and $\mathcal{H}_p$ denote the history fields associated with mechanics (tensile part of elastic strain density) and pore-pressure, respectively. The model departs from the traditional phase-field fracture models through the definition of the pressure history field $\mathcal{H}_p$. The physical constraints on damage ($0 \le d \le 1$; $\dot{d} \ge 0$) result in a specific form of the pressure history field $\mathcal{H}_p$ that must be implemented through a trial-state/return-mapping approach algorithmically.

3. RESULTS AND DISCUSSION

This section highlights the significance of the pressure history-field ($\mathcal{H}_p$) in simulating crack propagation in a poroelastic medium. We consider a square domain of size 1 m × 1 m with an initial pressure of 3 GPa, as shown in Fig. 1. All four boundaries are set as no-flow conditions to ensure an undrained condition. During the simulation, damage evolves uniformly throughout the domain until a crack initiates and propagates within the medium. Fig. 1 compares damage evolution over time between two scenarios: one with the pressure history-field ($\mathcal{H}_p$) and the other without this term. Evidently, $\mathcal{H}_p$ plays a crucial role in maintaining damage bounds and irreversibility. In particular, in the absence of the pressure history field, the damage becomes negative during the time interval from 0 to 0.01 seconds and, therefore, violates the following conditions: (a) $d \ge 0$, and (b) $\dot{d} \ge 0$.

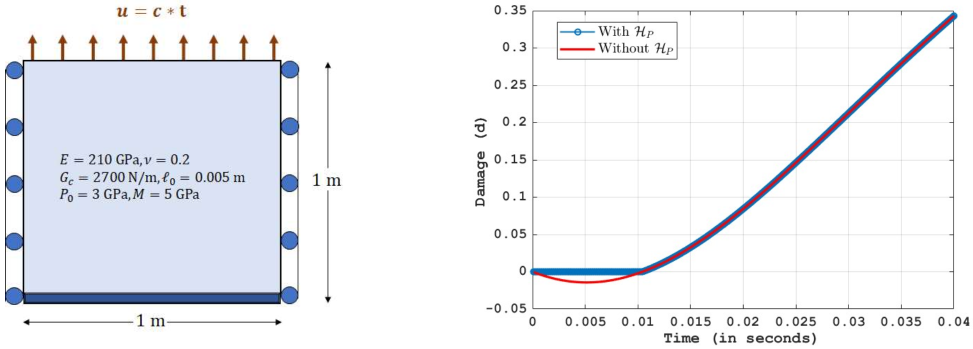

Figure 1: [Left] The schematic illustrating the geometry, boundary conditions, and material properties of the porous medium used to study the significance of $\mathcal{H}_p$ on damage evolution. [Right] The plot depicts the evolution of damage over time for two distinct scenarios, one with $\mathcal{H}_p$ and the other without $\mathcal{H}_p$.

REFERENCES

1. Q. Li, H. Xing, J. Liu, X. Liu, “A review on hydraulic fracturing of unconventional reservoir,” *Petroleum* **1**, pp. 8-15, 2015.
2. S. C. Cowin, “Bone poroelasticity,” *J. Biomechanics* **32**, pp. 217-238, 1999.
3. R. Duddu, J. N. Bassis, and H. Waisman, “A numerical investigation of surface crevasse propagation in glaciers using nonlocal continuum damage mechanics,” *Geophysical Res. letters* **40**, pp. 3064-3068, 2013.
4. M. Ambati, T. Gerasimov and L. D. Lorenzis, “A review on phase-field models of brittle fracture and a new fast hybrid formulation,” *Computational Mechanics* **55**, pp. 383–405, 2015.
5. C. Miehe, F. Welschinger and M. Hofacker, “Thermodynamically consistent phase-field models of fracture,” *Int. J. Num. Methods in Engg*. **83**, pp. 1273-1311, 2010.
6. M. A. Biot, “General Theory of Three-Dimensional Consolidation,” *J. of App. Physics* **12**, pp. 155-164, 1941.