Effect of magnetic field on non-miscible fluids flow through anisotropic porous medium

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Abstract: The flow of two immiscible, incompressible, electrically conducting viscous fluid of different viscosities through a porous channel in the presence of transverse magnetic field is investigated. The porous channel is homogeneous anisotropic structure. The flow is governed by generalized Brinkman-extended Darcy model. The continuity of shear stress, velocity is assumed at fluid-fluid interface, and no-slip condition at impermeable walls are used to obtained velocity profile. The strong influence of permeability ratio (K and κ), anisotropic angles (ϕ_1 and ϕ_2), Hartmann number (H_1 and H_2), and Darcy numbers ($Da_1$and Da_2) on velocity profile and flow rate are presented graphically and discussed.

1. INTRODUCTION & OBJECTIVE

The wide applications of immiscible fluid flow through porous medium in geophysical, in nature like in petroleum reservoir where oil passes through different types of pore structure like sand, rock, limestone, shale attracts many researchers to develop models like immiscible fluid flow through channel and other geometry [1]. A large number of studies in porous medium have been done by considering isotropic porous medium [1]. The key parameters that control permeability is inclination angle ϕ_1 and ϕ_2 and permeability ratio ($K = \frac{K_1}{K_2}$ and $\kappa = \frac{K_3}{K_4}$). Degan and Yovogan [2] studied the forced convective flow through horizontal porous channels of anisotropic in nature. Deo and Ansari [4] studied the effect of magnetic field on the two immiscible viscous fluids flow in a channel filled with porous medium. Recently, Karmakar and Raja Shekhar [5] used anisotropic angle to study the effect of anisotropic permeability on fluid flow through composite porous channel.

To evaluate the velocity profile of each layer, continuity of velocity, shear stress at the fluid-fluid interface is considered. The effect of key parameters of anisotropic porous medium, Hartmann numbers, Darcy numbers and viscosity ratio on velocity profile are graphically presented and discussed.

Mathematical formulation:

The physical model considered for this paper is illustrated in Fig. 1 consists the flow of two immiscible, incompressible, viscous fluid of different viscosity in separate porous layers of a horizontal channel. An external magnetic force is generated due to uniform strength of magnetic field is acted in transverse direction with negligible induced electric current and is given by $-\sigma_i B_o^2 u_i$ [3].

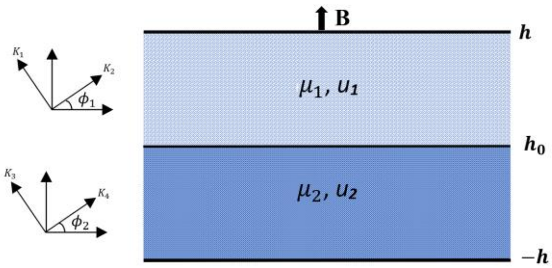

Fig.1 Model of the present problem

Region-I ($h_0 \le y \le h$):

$$\nabla \cdot \boldsymbol{V_1} = 0, \tag{1}$$

$$\boldsymbol{V_1} = \frac{K_{up}}{\mu_1}(-\nabla p_1 + \mu_{e1}\nabla^2 \boldsymbol{V_1} - \sigma_1 B_0^2 \boldsymbol{V_1}), \tag{2}$$

Region-II $(-h \leq y \leq h_0)$:

$$\nabla \cdot \boldsymbol{V_2} = 0, \quad (3)$$

$$\boldsymbol{V_2} = \frac{K_{lp}}{\mu_2}(-\nabla p_2 + \mu_{e2}\nabla^2 \boldsymbol{V}_2 - \sigma_2 B_0^2 \boldsymbol{V_2}), \quad (4)$$

where p_1 and $p_2$is the pressure, $\mu_1$andμ_2 is the viscosity of the fluid,$\mu_{e1}$and μ_{e2} is the effective viscosity, σ_1 and $\sigma_2$is the electrical conductivity and $K_{up}$and K_{lp} is the second-order symmetric permeability tensor of upper porous region and lower porous region, respectively and defined as

$$K_{up} = \begin{pmatrix} K_1 \sin^2 \phi_1 + K_2 \cos^2 \phi_1 & (K_2 - K_1) \sin \phi_1 \cos \phi_1 \\ (K_2 - K_1) \sin \phi_1 \cos \phi_1 & K_2 \sin^2 \phi_1 + K_1 \cos^2 \phi_1 \end{pmatrix} \quad (5)$$

and

$$K_{lp} = \begin{pmatrix} K_3 \sin^2 \phi_2 + K_4 \cos^2 \phi_2 & (K_4 - K_3) \sin \phi_2 \cos \phi_2 \\ (K_4 - K_3) \sin \phi_2 \cos \phi_2 & K_4 \sin^2 \phi_2 + K_3 \cos^2 \phi_2 \end{pmatrix}. \quad (6)$$

Solution:

Applying the following dimensionless parameters,

$$x^* = \frac{x}{h}, y^* = \frac{y}{h}, P_0 = \frac{V_0 \mu_1}{h}, P^* = \frac{P}{P_0}, u_i^* = \frac{u_i}{V_0}, \gamma = \frac{\sigma_1}{\sigma_2}, \alpha = \frac{\mu_1}{\mu_2}, \lambda_i = \frac{\mu_{ei}}{\mu_i}, i = 1,2 \quad (7)$$

in the above governing equations.

Dimensionless boundary conditions are given as:

1. At $y = 1, u_1 = 0$ (10)
2. At $y = 0, u_1 = u_2$ (11)
3. At $y = 0, \sigma T_{xy}^{(1)} = T_{xy}^{(2)}$ (12)
4. At $y = -1, u_2 = 0$ (13)

2. Results & Highlights of important points

The effect of different parameters anisotropic ratio(K, κ), anisotropic angles (ϕ_1, ϕ_2), conductivity ratio (γ),Hartmann numbers (H_1),viscosity ratio (α), and Darcy numbers (Da_1, Da_2) on velocity profile of immiscible, electrically conducting, incompressible viscous fluid flow through two different porous layers studied in this paper. The following observation are made:

1. Increasing the anisotropic ratio, anisotropic angle and Hartmann number decreased the velocity profile.
2. Increasing the viscosity ratio, Darcy number and conductivity ratio increased the velocity profile.

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