An implicit numerical approach to solve Fisher's equation using modified B-splines

Liju P[1a] and A. Awasthi[1b]

[a] Research Scholar, National Institute of Technology Calicut, Kerala

[b] National Institute of Technology Calicut, Kerala

1. INTRODUCTION & OBJECTIVE

In this paper, we propose an effective numerical method to solve a non-linear reaction-diffusion equation of Fisher-type. This equation is widely used in the different branches of sciences and engineering. In particular, the solution of Fisher equation helps to find growth of tumor cells in the brain. Here, we use a numerical strategy based on modified cubic B-splines for spatial variable and its derivative, and backward differentiation formulas for temporal variable. Proposed numerical method is implicit in nature and unconditionally stable. Validation of the method is carried out by calculating the rate of convergence and illustrated by test examples.

We consider the reaction - diffusion equation of Fisher-type

$$u_t = \lambda u_{xx} + \mu u(1-u), \qquad a \le x \le b,\ t > 0,$$

with the initial and boundary conditions

$$u(x,0) = u_0(x), \qquad a \le x \le b,$$

$$u(a,t) = f_0(t), \qquad u(b,t) = f_2(t), \qquad t \in [0,t].$$

This paper deals the Fisher equation numerically by using modified form of B-splines and backward differentiation formulas. The backward differentiation formula is an implicit multi step numerical method that is used to solve a system of first order ordinary differential equations. This system of differential equations is a result of implementation of modified basis spline on space variable and on spatial derivatives. The solution $u(x,t)$ of fisher equation is approximated by the linear combination of modified cubic B-spline basis functions $\beta_j(t)$ follows.

$$u(x,t) = \sum_{j=0}^{n} \alpha_j(t)\beta_j(t)$$

Where $\beta_0(x) = \beta_0{}'(x) + 2\beta'_{-1}(x)$, $\beta_1 = \beta'_1(x) - \beta'_{-1}(x)$

$\beta_j(x) = \beta'_j, j = 2,3,\dots n$, $\quad \beta_{n-1}(x) = \beta'_{n-1}(x) - \beta'_{n+1}(x)$, $\quad \beta_n(x) = \beta_n{}'(x) + 2\beta'{}_{n+1}(x)$

Here $\beta'_j,\ j = -1,0,1,\dots n, n+1$ be the cubic B-splines at the knots x_j for the mesh $a = x_0 < x_1 < \cdots x_n = b$ with step size $h = \frac{b-a}{n}$ and $\alpha_j(t)$ are the coefficients.

The approximate values of $u(x,t)$ and its derivatives at the knots are determined as follows,

$$u(x_j,t) = \alpha_{j-1}(t) + 4\alpha_j(t) + \alpha_{j+1}(t)$$

$$u_x(x_j,t) = \frac{3}{h}(\alpha_{j+1}(t) - \alpha_{j-1}(t))$$

$$u_{xx}(x_j,t) = \frac{6}{h^2}(\alpha_{j-1}(t) - 2\alpha_j(t) + \alpha_{j+1}(t))$$

$$u_t(x_j,t) = \alpha'_{j-1}(t) + 4\alpha'_j(t) + \alpha'_{j+1}(t) \qquad j = 1,2,\dots n-1$$

$$\boldsymbol{u_t(0,t) = 6\alpha'_0 = f'_0(t)} \qquad \qquad \boldsymbol{u_t(n,t) = 6\alpha'_0 = f'_1(t)}$$

We get a system of ordinary differential equations of the form

$$\boldsymbol{A\frac{d\alpha}{dt} = \psi(\alpha).}$$

Now we solve the system by backward differentiation formula

$$\boldsymbol{A\alpha^{i+1} = \frac{4}{3}A\alpha^{i} - \frac{1}{3}A\alpha^{i-1} + \frac{2\Delta t}{3}\psi(\alpha^{i+1})}$$

2. RESULTS & HIGHLIGHTS OF IMPOINTANT POINTS

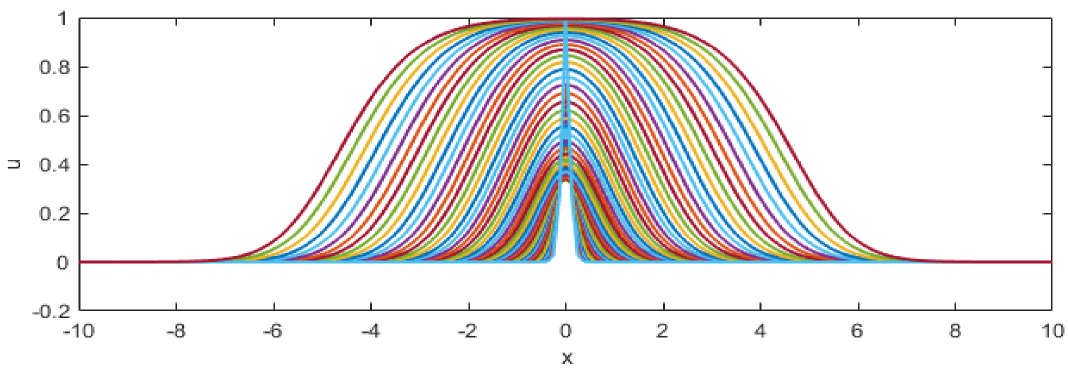
The Fisher equation is a classic example of a partial differential equation which nonlinear in nature and have lots of applications in sciences and engineering including medical science.

It has been observed that after semi discretization by modified B splines, it leads to a system of nonlinear ordinary differential equations, which exhibits stiffness. Therefore, a need of stiff solver arises, to circumvent this we use backward differentiation formula, which is implicit in nature and incorporate stiffness.

Most of existing methods are based on explicit and non-stiff solvers, therefore, there is high expectation to use a suitable stiff solver which can work well for all values of λ, a diffusion coefficient. The primary results of the work are appended below.

We examine the test example using to demonstrate the method's adaptability with $\lambda = 0.1,\ \beta = 1$ and initial condition $u_0(x) = sech^2(10x)$ and boundary conditions $\lim_{x\to-\infty} u(x,t) = \lim_{x\to\infty} u(x,t) = 0$.

We set $a = -10, b = 10, n = 160$ and $dt = 0.01$, the $u - x$ plots of different time are shown below



REFERENCES

1. R.C. Mittal, R.K. Jain, Numerical *solutions of nonlinear Fisher's reaction-diffusion equation with modified cubic B-splines collocation method,* Mathematical Science, Springer, 2013.
2. R. Rohila, R.C. Mittal, *Numerical study of reaction diffusion Fisher's equation by fourth order cubic B-spline collocation method*, Springer, 2018.
3. R.C. Mittal, G. Arora, *Efficient numerical solution of Fisher equation by using B-spline method,* International journal of computational mathematics, Taylor & Francis, *2010*.
4. K. Swanson, R. Rostomily, E. Alvord, *A mathematical modelling tool for predicting survival of individual patients following resection of glioblastoma: a proof of principle*, British Journal of Cancer, 2008.