On the motion of a micropolar fluid drop in a micropolar fluid

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Abstract

The Stokes axisymmetric flow of an incompressible micropolar fluid past a micropolar fluid sphere is studied analytically. The boundary conditions used are the vanishing of the normal velocities, the continuity of the tangential velocities, continuity of shear stresses and zero microrotation at the surface of the drop. The hydrodynamic drag force acting on the drop is calculated. Well known results are reduced and comparisons are made with classical viscous-viscous droplet, micropolar-viscous droplet and viscous-micropolar droplet.

Introduction and Objective

The motion of a fluid droplet in a second immiscible fluid through a continuous medium at low Reynolds numbers is of much interest in the fields of chemical, biomedical, and environmental engineering and science. This study plays an important role in natural and industrial processes such as raindrop formation, the mechanics and rheology of emulsions, liquid-liquid extraction, motion of blood cells in an artery or vein, extraction of crude oil from petroleum products and sedimentation phenomena. The creeping flow motion of a single spherical drop in an unbounded medium was first analyzed independently in [1,2]. A well accepted theory which accounts for internal structures of fluids is micropolar fluid theory by Eringen [3]. These fluids consist of rigid, randomly oriented particles with their own spins and micro rotations suspended in a viscous medium. The problems of the flow of a viscous-micropolar, micropolar-viscous studied by different researchers [4-7]. The Stokes flow of micropolar fluid past a rigid sphere is investigated by [8,9].

The main object of the paper is to calculate the resistant force exerted on a micropolar fluid sphere moving with a uniform velocity in an unbounded micropolar fluid. To the best of author's knowledge the current investigation is not done earlier. The variation of drag force versus viscosity ratio and micropolarity parameters are presented graphically and discussed. Some previously published well-known results are also deduced from the present analysis.

Problem formulation

Consider a micropolar fluid sphere moving into an unbounded volume of another immiscible micropolar fluid. The following assumptions are considered the fluid inside the particle is micropolar fluid and the fluid in the surrounding medium is micropolar fluid, the flows are steady, axisymmetric, there is no interfacial mass transfer (the radial velocity is zero at interface),there are no surface-active materials.

Let the steady axisymmetric flow of an incompressible micropolar fluid past a micropolor fluid sphere which is held fixed in a uniform stream of velocity U. The external region and the internal region are denoted by regions I and II respectively. The equations of motion for the region outside the sphere are the equations governing the steady flow of an incompressible micropolar fluid under Stokesian assumption with the absence of body force and body couple and are given by [3]

$$\nabla \cdot \mathbf{q}^{(i)} = 0, \tag{1}$$

$$\nabla p^{(i)} + (\mu_i + \kappa_i)\nabla \times \nabla \times \mathbf{q}^{(i)} - \kappa_i \nabla \times \mathbf{v}^{(i)} = 0, \tag{2}$$

$$\kappa_i \nabla \times \mathbf{q}^{(i)} - 2\kappa_i \mathbf{v}^{(i)} - \gamma_i \nabla \times \nabla \times \mathbf{v}^{(i)} + (\alpha_i + \beta_i + \gamma_i)\nabla(\nabla \cdot \mathbf{v}^{(i)}) = 0, i = 1,2. \tag{3}$$

where $\mathbf{q}^{(i)}, \mathbf{v}^{(i)}$, and $p^{(i)}$ are velocity vector, microrotation vector and pressure, μ_i is the viscosity coefficient of the classical viscous fluid and κ_i is the rotational viscosity coefficient. α_i, β_i and γ_i are the gyroviscosity coefficients for the micropolar fluids.

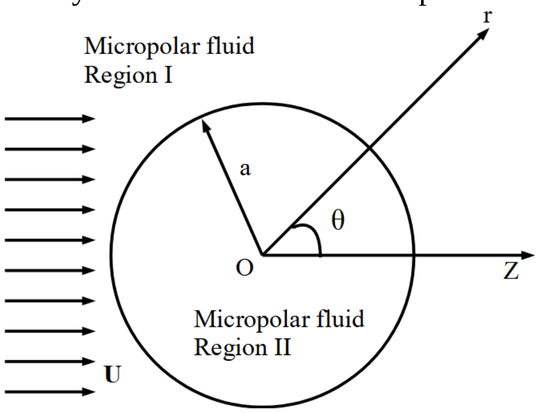

Figure1: Geometry of the problem

Conclusions

An analytical solution for the problem of Stokes flow of micropolar fluid past a micropolar fluid sphere is obtained. The drag force is calculated and the dependence of the drag force on the micropolarity parameters χ_1, χ_2 and the classical viscosity ratio σ is depicted graphically.

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