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**Surface effects on dispersion behaviors of SH waves in piezoelectric-piezomagnetic bilayer plates under external magnetic field using nonlocal theory of elasticity**

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**ABSTRACT**

**Research aim:** This study investigates the nonlocal and surface effects on the dispersion behaviors of SH waves in PE-PM bilayer plates under an external magnetic field. Based on Eringen's nonlocal elasticity theory, constitutive relations and equations of motion are developed by adding an inherent length. The generalized Young-Laplace equations and the G-M model have been utilized to include surface effects in the boundary conditions. The closed-form dispersion equation has been obtained analytically under different external magnetic fields for electrically open and magnetically short conditions. The analytical and numerical results show that the SH wave dispersion curves are highly influenced by surface stress, nonlocal scale parameter, and external magnetic fields. This study provides valuable insights into complex wave dynamics, helping to optimize the performance and functionality of such smart composites in various engineering applications.

**Literature survey:** In the realm of nanotechnology and advanced materials, the study of wave propagation holds immense importance for understanding and harnessing the mechanical behavior of plates at the nanoscale due to their widespread applications in various engineering fields. The mathematical framework for the surface theory of elasticity was formulated by Gurtin and Murdoch ([1], [2]). To account for nanoscale effects in nano-structures, Eringen [3] developed one of the well-known theories, the nonlocal elasticity theory for nanoscale analysis. Recently, Nath and Dhua ([4]-[5]) investigated shear wave scattering in orthotropic inhomogeneous magneto-visco-elastic multi-layered structure on the influence of initial stress. Through theoretical modeling and numerical simulations, this article aims to uncover the intricate interplay between surface effects, nonlocal behaviors, and wave propagation on PE-PM bilayer systems under different external magnetic fields.

**Problem formulation:** The problem to be considered is depicted in Fig. 1, with piezoelectric and piezomagnetic bilayer plates of thickness  $h_1$  and  $h_2$ , respectively. Both media are considered to be transversely isotropic. The top and bottom surfaces are treated as the upper and bottom surface layers, respectively. The constitutive relations for the nonlocal model can be written as:

$$L\bar{\sigma}_{ij} = \sigma_{ij} \quad (1)$$

$$L\bar{D}_i = D_i \quad (2)$$

$$L\bar{B}_i = B_i \quad (3)$$

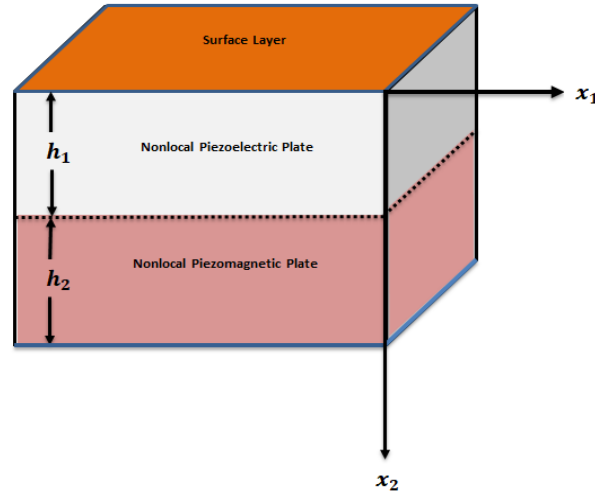
Moreover, The nonlocal dynamic equations of motion can be written as;

$$\sigma_{ij,j} = \rho L \ddot{u}_i \quad (4)$$

$$D_{i,i} = 0, \quad (5)$$

$$B_{i,i} = 0. \quad (6)$$

Where,  $\bar{\sigma}_{ij}$ ,  $\bar{D}_i$ ,  $\bar{B}_i$  are the nonlocal stress, nonlocal electric displacement, and nonlocal magnetic flux respectively, and the linear differential operator  $L = 1 - \varepsilon^2 \nabla^2$ , where  $\varepsilon$  is the nonlocality parameter.



a) Non-classical Boundary Conditions for the upper and bottom surfaces:

$$\begin{aligned}\bar{\sigma}_{31,1}^{(sk)} - \bar{\sigma}_{32}^{(k)} &= \rho^{(sk)} u_3^{(k)}, \\ \bar{D}_{1,1}^{(sk)} - \bar{D}_2^{(k)} &= 0, \\ \bar{B}_{1,1}^{(sk)} - \bar{B}_2^{(k)} &= 0,\end{aligned}\quad (7)$$

b) Also, across the interface the electric field, magnetic field, and stress are given below:

$$\begin{aligned}\varphi^{(1)}(x_1, 0) &= \varphi^{(2)}(x_1, 0), \\ \psi^{(1)}(x_1, 0) &= \psi^{(2)}(x_1, 0), \\ (1 - \epsilon^2 \nabla^2) \bar{D}^{(1)}(x_1, 0) &= (1 - \epsilon^2 \nabla^2) \bar{D}^{(2)}(x_1, 0), \\ (1 - \epsilon^2 \nabla^2) \bar{B}^{(1)}(x_1, 0) &= (1 - \epsilon^2 \nabla^2) \bar{B}^{(2)}(x_1, 0), \\ (1 - \epsilon^2 \nabla^2) \bar{\sigma}_{32}^{(1)}(x_1, 0) &= (1 - \epsilon^2 \nabla^2) \bar{\sigma}_{32}^{(2)}(x_1, 0) = \alpha \left[ u_3^{(1)}(x_1, 0) - u_3^{(2)}(x_1, 0) \right]\end{aligned}\quad (8)$$

**Significant conclusions:** In conclusion, the study of nonlocal effects and surface effects significantly influence the dispersion properties, including the phase velocity of shear horizontal waves in the bilayer system. The presence of nonlocal effects further enhances the complexity of wave propagation by introducing long-range interactions and spatial dependencies. On the other hand, surface effects in a nonlocal solid can have significant physical implications such as surface characteristics exhibiting size-dependent dispersion behaviors on the propagation of the shear wave in these bilayer plates. But, the coupling influence of surface piezoelectricity and nonlocal small-scale is more significant in comparison with individual effects.

**Keywords:** Surface theory of elasticity, Nonlocal elasticity, Piezoelectric-piezomagnetic material, Plate model, SH-wave.

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