An effective analytical approach for time fractional Fokker-Planck equations

Neetu Garg[a] and Varsha R[b]
a, b Department of Mathematics,
National Institute of Technology Calicut,
673601, India

1. INTRODUCTION & OBJECTIVE

The area of fractional calculus is attaining wide growth nowadays. It deals with the integrals and derivatives of arbitrary order. Many fields like fluid dynamics, control theory, biological modelling and more are adapting concepts and applications from fractional calculus. Several real world problems can be precisely modelled by fractional differential equations.

Fokker and Planck first proposed the Fokker-Planck equations which depict the Brownian motion of particles and the probability change of a random function in time and space. The fractional model of the Fokker-Planck equation has numerous applications in physical problems such as the Brownian movement of particles, anomalous diffusion, polymer dynamics, and so on [1]. This work is concerned with the time fractional Fokker-Planck equations (TFFPE) having the general form

$$D_t^{\alpha} u(x,t) = \left[-\frac{\partial}{\partial x} f_1(x,t) + \frac{\partial^2}{\partial x^2} f_2(x,t)\right] u(x,t), \quad x, t \geq 0, \; 0 \leq \alpha \leq 1$$

with initial condition $u(x, 0) = \mathrm{y}(x)$, where y is a smooth function. And f_1 and f_2 are drift coefficients and diffusion coefficients, respectively. Here α is the order of the Caputo fractional derivative. For $\alpha = 1$, the TFFPE reduces to the classical Fokker-Planck equation.

In past years, several analytical and numerical methods have been utilized to solve classical and fractional model Fokker-Planck equations including the Yang transform decomposition method [1], shifted Chebyshev collocation method [2], iterative Laplace transform method [3], Adomian's decomposition method [4], etc.

This work aims to find an analytical solution for TFFPE using the Laplace Residual Power Series (LRPS) method. The proposed method combines Laplace Transform (LT) and Residual Power Series (RPS) methods. The main point of the LRPS method is to apply LT to a fractional differential equation and use the fractional power series expansion as the solution of the transformed equation. Similar to the RPS method, we are using the concept of limit at infinity to determine the coefficients in the series expansion. Finally, we apply inverse LT to obtain the series solution of the concerned fractional differential equation.

In 2020, T. Eriqat et al. first suggested the LRPS method for solving neutral fractional pantograph equations [5]. This method has acquired attention for finding solutions to different types of fractional differential equations like time fractional dispersive PDEs [6], Burger's system of nonlinear fractional PDEs [7], and so on.

2. RESULTS & HIGHLIGHTS

In this work, TFFPE has been solved analytically using the LRPS method, which is a combination of LT and RPS method. Application of LT on TFFPE equations transforms the fractional differential equation into a simple algebraic equation. The advantage of this method over the RPS method is the straightforward computations for determining unknown coefficients. The results extracted from the LRPS method and other methods are compared and found to be in good agreement. The obtained solutions have been displayed graphically at different fractional orders $\alpha \in (0,1]$. The obtained series expansion gives the accurate approximate solution of TFFPE equations. Thus, the LRPS method is very simple and efficient for finding approximate analytical series solutions to nonlinear fractional differential equations.

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