Unsteady Flow past an Infinite Vertical Cylinder in Rotating Fluid under Influence of Vertical Magnetic Field

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Abstract

This paper presents the analytical solution of unsteady natural convective flow of a viscous incompressible fluid past on an isothermally heated infinite vertical cylinder. Uniform magnetic field is applied in the direction perpendicular to the cylinder and normal to the direction of the flow. Considering that the pressure is uniform in the whole flow field, the closed-form solutions of the dimensionless unsteady coupled linear governing boundary layer equations are obtained in terms of Bessel functions and modified Bessel functions by Laplace transform method. Graphically analyzed the velocity profiles and skin-friction profiles for different values of Ekman number are analyzed along with the temperature profiles and Nussult number. Response surface methodology is applied for the existing data sequence of Nusselt number so as to find its optimal response. The regression coefficients, in the equation of fitted line response data, are considered until third order for achieving a more accurate solution. The significance of values of regression coefficients is analysed based on the analysis of variance (ANOVA) method.

LITERATURE SURVEY

Magnetohydrodynamic rotating natural convective fluid flow over a vertical cylinder plays an important role in the applications of geophysical problems. Sparrow and Gregg [1] first studied laminar free convection heat transfer from the outer surface of a vertical circular cylinder. They presented analytical solutions using Laplace transform technique. Greenspan and Howard [2] initiated the study of time-dependent motion of a rotating fluid. They found that the development of the Ekman layer, and it is concluded that the Ekman layer plays the significant role in the transient flow. After these studies, there have been many works on the dynamics of spin-up and spin-down of a homogeneous rotating stratified fluid. Goldstein and Briggs [3] studied and analyzed the transient free convection flow with heat transfer problem from vertical circular cylinders to a surrounding initially quiescent fluid by using the Laplace transform technique. Ganesan and Rani [5] investigated numerical solution for transient natural convection flow over a vertical cylinder with heat and mass transfer. Ganesan and Rani [6] studied magnetohydrodynamic effect on flow past a vertical cylinder with heat and mass transfer. Ganesan and Loganathan [7] studied the effect of the magnetic field on a moving vertical cylinder with constant heat flux numerically. They observed that the presence as well as increase in the magnetic field leads to decrease in the velocity field and rise in thermal boundary thickness. Rudra kanta Deka and Ashish Paul [9, 10] have investigated unsteady natural convective flow past an infinite vertical cylinder with constant surface temperature using analytical method.

MATHEMATICAL FORMULATION

We have considered an unsteady homogeneous laminar incompressible and electrically conducting viscous fluid flow past an infinite vertical cylinder of radius r_0. Cylindrical coordinate system is assumed. The x-axis is taken along the axis of the cylinder in the vertically upward direction, from the leading edge of the cylinder and the radial coordinate r is taken normal to it. The flow quantities depend on r and t only, because of the vertical homogeneity (axially symmetric) of the flow problem. A uniform magnetic field β_0 is applied in the direction perpendicular to the horizontal fluid flow system. The fluid is assumed to be slightly conducting so that the magnetic Reynolds number << 1, and hence the induced magnetic field is negligible in compared with the imposed magnetic field. The uniform pressure is assumed in the whole flow field. Analytically the exact solution for the velocity profile has been obtained and the structures of the associated Ekman-Hartmann boundary layer have been analyzed. With the above assumptions, the equations of motion for the unsteady flow are as follows:

$$\frac{\partial(ur)}{\partial x} + \frac{\partial(vr)}{\partial r} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t'} - 2\omega v = g\beta(T' - T'_\infty) + \frac{\gamma}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{\sigma\beta_0^2}{\rho}u \quad (2)$$

$$\frac{\partial v}{\partial t'} + 2\omega u = \frac{\gamma}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) - \frac{\sigma\beta_0^2}{\rho}v \quad (3)$$

$$\frac{\partial T'}{\partial t'} = \frac{\alpha}{\rho c_p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T'}{\partial r}\right) \quad (4)$$

The initial and boundary conditions are

$$\left.\begin{aligned} &t' \le 0: u = 0, v = 0, T' = T'_\infty, \quad \forall\, r \\ &t' > 0: u = 0, v = 0, T' = T'_w,\ at\ r = r_0 \\ &u \to 0, v \to 0, T' \to 0\ as\ r \to \infty \end{aligned}\right\} \quad (5)$$

Equations (2) and (3) can be rewritten as a single equation using the relationship $w = u + i\,v$,

$$\frac{\partial w}{\partial t'} + 2i\omega w = g\beta(T' - T'_\infty) + \frac{\gamma}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) - \frac{\sigma\beta_0^2}{\rho}w \quad (6)$$

Introducing the non-dimensional quantities, $R = \frac{r}{r_0}, t = \frac{t'\gamma}{r_0^2}, W = w\frac{r_0}{\gamma}, T = \frac{T' - T'_\infty}{T'_w - T'_\infty}$

Equations (1), (4) and (6) reduces to,

$$\frac{\partial(RU)}{\partial X} + \frac{\partial(RV)}{\partial R} = 0 \quad (7)$$

$$\frac{\partial T}{\partial t} = \frac{1}{\Pr R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) \quad (8)$$

$$\frac{\partial W}{\partial t} + \frac{2i}{Ek}W = Gr\,T + \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial W}{\partial R}\right) - MW \quad (9)$$

where u axial velocity, v transverse velocity, T' temperature of the fluid, r radius of the cylinder, U dimensionless axial velocity, V dimensionless transverse velocity, T dimensionless temperature, R dimensionless radius, g acceleration due to gravity, ν kinematic viscosity, β coefficient of volume expansion, M magnetic parameter, κ magnetic diffusivity, η thermal conductivity, ρ density, C_p specific heat at constant pressure T'_∞ ambient temperature, E Ekman number, Pr Prandtl number and Gr Grashof number.

The non-dimensional form of initial and boundary conditions is given as:

$$\left.\begin{aligned} &t \le 0: W = 0, T = 0, \quad \forall\, R \\ &t > 0: W = 0, T = 1,\ at\ R = 1 \\ &W \to 0, T \to 0 \quad as\ R \to \infty \end{aligned}\right\} \quad (10)$$

The analytical solution of the momentum equation and heat equations are derived by the Laplace transform technique.

CONCLUSIONS

In order to get an insight of the physics of the problem, the numerical results are presented to outline the general physics involved in the effects of different *M* and *Ek* with fixed *Pr* and *Gr* on the axial and transient velocity and temperature profiles. The physical behaviour of non-dimensional velocity, skin-friction, temperature and Nussult number (the coefficient of heat transfer rate) are discussed graphically. Increasing values of *Ek* and *M*, the velocity profiles and skin-friction are shown graphically. When *Ek* increases the velocity vector is increases. For the higher *Pr* values, the temperature profiles and Nussult number are analyzed. It is observed that the heat transfer rate is more when *Pr* increases. Due to the effect of uniform magnetic field on rotating flow over vertical cylinder, the axial velocity profiles are increases and transient velocity profiles decreases with respect to radius of the cylinder. Also, an attempt is made to derive the regression equations for calculating rate of heat transfer as a function of other parameters. The regression coefficients involved in the equation are tested using the ANOVA statistical tool. The optimum value of rate of heat transfer is obtained without numerical calculations.

References:

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