Stochastic Projection-based Physics Informed Neural Network for Solving PDEs

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1. INTRODUCTION & OBJECTIVE

The current form of PINN was introduced and termed by Raissi et al. [1], whereas its fundamental concept has existed since the early 1990s [2]. Contrary to data- driven approaches that rely solely on output data for training, Physics-Informed Learning (PIL) frameworks incorporate the governing physics to the loss function during the training process. Even though PINN excels in solving forward and inverse problems, there are signicant bottlenecks associated with it [3]. Often the vanilla PINN fails to obtain desired results. To that end, we propose a framework, namely a stochastic projection-based physics-informed neural network (SP-PINN). The proposed approach not only eliminates the need forautomatic differentiation but also improves the prediction accuracy of the PINN, especially for problems involving irregular domains, complex solution domains, and domains with discontinuities.

The primary objective of this study is to develop the Stochastic Projection-based Physics-Informed Neural Network (SP-PINN) and demonstrate its efficacy in solving Partial Differential Equations (PDEs). We consider the residual formulation of general parameterized nonlinear partial differential equations for the given initial and boundary conditions, given by:

$$\mathcal{N}(\boldsymbol{x}, t, \partial_{\mathbf{t}}, \partial_{\mathbf{t}}^2 \ldots, \partial_x, \partial_t^n, \ldots, \partial_x^n, \boldsymbol{\alpha})u = 0,\ \boldsymbol{x} \in \Omega, t \in [0, T]$$

Here, the space and time coordinated are denoted as $x \in \mathbb{R}^d$. $\mathcal{N}$ in Eq. (1) denotes the nonlinear operator corresponding to a given governing physics PDE, whereas α represents the system parameters. To solve the given PDE Eq.(1), the vanilla PINN [1] computes the residual loss function, employing automatic differentiation, which is minimized to obtain the solution. However, to obtain the gradients in the residual function here, we employ stochastic projection theory [4], where the gradients are given by:

$$\mathbf{G}(\boldsymbol{X} = \overline{\boldsymbol{X}}) = \frac{\partial u}{\partial \boldsymbol{X}} = \frac{\frac{1}{N_t}\sum_{i=1}^{N_b}\ (u - \bar{u})(\boldsymbol{X}_i - \overline{\boldsymbol{X}})^T}{\frac{1}{N_t}\sum_{i=1}^{N_b}\ (\boldsymbol{X}_i - \overline{\boldsymbol{X}})(\boldsymbol{X}_i - \overline{\boldsymbol{X}})^T}$$

2. RESULTS & HIGHLIGHTS OF IMPOINTANT POINTS

For the numerical illustration, we consider the example of Poisson's equation over an L-shaped domain. The governing equation for the problem is given by:

$$u_{xx} + u_{yy} = 1, \qquad x \in [-1, 1], y \in [-1, 1]$$

The prediction results of the SP-PINN are presented in Fig. 1 and the table.1

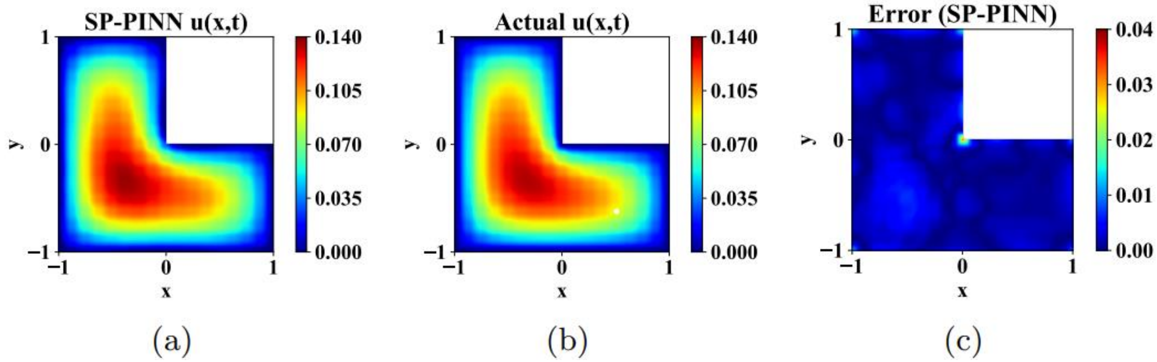

Fig. 1. Results of Poisson equation on L-shaped domain; a) Solution predicted by the SP-PINN, (b) Actual solution (c) Error

Method	Number of collocation points	Max. absolute error	Average error
SP-PINN	1935	0.03	0.002
AD-PINN	1935	0.04	0.003

Table.1 Comparison of prediction error for Poisson's example on L-shaped domain

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