Uncertain Dynamic Response Analysis of Fractionally Damped Beams Subjected to Various External Loads using Fuzzy Laplace Homotopy Perturbation Analysis (FLHPA)

Anapagaddi Sudhir Kumar[a,b] and Sudarshan Dhua[b]
[a b]Department of Mathematics, Rajiv Gandhi University of Knowledge Technologies, Nuzvid, Andhra Pradesh, India,
[b]Department of Mathematics, School of Sciences, National Institute of Technology Andhra Pradesh, India,

[a]E-mail: a.sudhirk@yahoo.com, [b]Email: dhuasudarshan@gmail.com

1. INTRODUCTION & OBJECTIVE

Abstract: In this article Fuzzy Laplace Homotopy Perturbation Analysis (FLHPA)method is used to solve fuzzy fractional equations in a beam problem. To determine the uncertain dynamic responses analysis of the fractionally damped beam, FLHPA has been successfully implemented along with triangular fuzzy numbers and Gaussian fuzzy numbers. The uncertain responses subject to impulse and step loads have also been computed and the behaviors of the responses are analyzed.

Literature survey: Fractional differential equations have recently been employed to model a variety of physical and engineering problems. Almeida et al.'s [1] great explanation of the benefits of fractional order derivatives when taking differential equations into account. They have highlighted that fractional differential equations are more effective in simulating a variety of issues with high-order dynamics and complex nonlinear processes. The related parameters and variables in the existing investigations are typically well defined, however in actual practice there may be uncertainty due to observational error, maintenance-induced error, etc.

However, many researchers have created numerous approaches to solving physical systems' integral equations as well as ordinary and partial differential equations of any order. In some cases, it may be difficult to obtain the analytical solution for these kinds of differential equations, necessitating the use of a trustworthy and effective numerical technique. Behera and Chakraverty [2,3] recently investigated the homotopy perturbation method for the numerical solution of a fractionally damped beam. Monami and Odibat [4,5,6] demonstrated that the homotopy perturbation technique (HPM) is an alternative analytical method for fractional differential equations by applying it to them along with a Laplace transform. Recently, Satsanit, Wanchak [7], solved linear and non linear fractional beam equations by using crisp methods.

2. RESULTS & HIGHLIGHTS OF IMPOINTANT POINTS

In the present analysis,the fractional derivatives are considered in sense of Caputo and FLHPA is developed. For the numerical solution of the uncertain dynamic response of a fuzzy fractionally damped beam with fuzzy initial condition in the current analysis, FLHPA is used. TFN and GFN are used to define uncertainty in the initial condition. For the purposes of this analysis, unit step and impulse loads are taken into account.

Solution to uncertain fractionally damped viscoelastic beam:

To develop numerical schemes for a fuzzy fractionally damped viscoelastic beam, let us consider a fuzzy linear differential equation which describes the dynamics of the system with the damping as an arbitrary fractional derivative of order α:

$$\rho A \frac{\partial^2 \widetilde{U}}{\partial t^2} + c \frac{\partial^\alpha \widetilde{U}}{\partial t^\alpha} + EI \frac{\partial^4 \widetilde{U}}{\partial x^4} = F(x,t)$$

where ρ, A, c, E and I represents the mass density, cross-sectional area, damping coefficients per unit length, Young's modulus of elasticity and moment of inertia of the beam, respectively. $F(x,t)$ is the externally applied force and $\widetilde{U}(x,t)$ is the transverse fuzzy displacement. $\frac{\partial^\alpha}{\partial t^\alpha}$ is the fractional derivative of order $\alpha \epsilon [0,1]$ of the fuzzy displacement function $\widetilde{U}(x,t)$. Initial conditions are considered as fuzzy viz $\widetilde{U}(0) = \widetilde{U}'(0) = \widetilde{K}$, Where $\widetilde{K}$ be consider as two cases (i) TFN and (ii) GFN.Max and Min of r^{th}- cut are considered based on the type of differentiability and solved by using FLHPA.

Conclusion:

FLHPA has developed and successfully been applied to obtain the uncertain dynamic responses of fuzzyfractionally damped simply supported beam using TFN and GFN. It was found that this method obtained the rapid convergence of the series solution. There is no effect in the solution based on fuzzy number type, so it is concluded that taking GFN gives good impact on the solution as most of the uncertainties in nature can be described by Gaussian distribution. Furthermore, FLHPA does not require any discretization, which enables us to get around the challenges posed by round-off errors, high computer memory requirements, and repeated calculations. The findings demonstrate that the FLHPA is an effective and strong technique for resolving fractional fuzzy PDEs.

REFERENCES

[1] Almeida, R., Bastos, N. R., & Monteiro, M. T. T. (2016). Modeling some real phenomena by fractional differential equations. Mathematical Methods in the Applied Sciences, 39(16), 4846-4855.

[2] Chakraverty, S., & Behera, D. (2013). Dynamic responses of fractionally damped mechanical system using homotopy perturbation method. Alexandria Engineering Journal, 52(3), 557-562.

[3] Chakraverty, S., & Behera, D. (2015). Uncertain dynamic responses of fuzzy fractionally damped spring-mass system. Journal of Intelligent & Fuzzy Systems, 29(1), 327-336.

[4] Momani, S. (2005). Analytical approximate solution for fractional heat-like and wave-like equations with variable coefficients using the decomposition method. Applied Mathematics and Computation, 165(2), 459-472.

[5] Momani, S., &Odibat, Z. (2007). Comparison between the homotopy perturbation method and the variational iteration method for linear fractional partial differential equations. Computers & Mathematics with Applications, 54(7-8), 910-919.

[6] Momani, S., &Odibat, Z. (2007). Comparison between the homotopy perturbation method and the variational iteration method for linear fractional partial differential equations. Computers & Mathematics with Applications, 54(7-8), 910-919.

[7] Satsanit, W. (2021). On the solution linear and nonlinear fractional beam equation. Journal of Nonlinear Sciences & Applications (JNSA), 14(3).