Predictive and Interpretable digital twin for dynamical systems

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1. INTRODUCTION & OBJECTIVE

A digital twin (DT) can be idealized as a virtual shadow of the underlying physical system [1]. Due to recent innovations in Industry 4.0, cloud computing, and computer-aided modeling, the DT has received strategic priorities to improve the performance of physical systems. However, existing DTs assume that the physics of the underlying dynamical system does not evolve with time, and one can track the evolution of the system dynamics by estimating the perturbation in the system parameters [2]. This assumption has minimal applications since the physics of all dynamical systems evolve in time; for example, consider a degrading dynamical system, where the physics changes on a slow-time scale due to fatigue. If the physics of the mirror model is not updated, the DT as a whole will be un-synchronized from the physical twin, defeating the purpose of devising the DT. Modern data-driven machine learning (ML) methods may provide a solution [3]; however, they are difficult to interpret, and their long-term generalization is not guaranteed. In this work, we propose a framework for updating the physics of mirror models using physics-based interpretable functions.

In particular, we construct a library of various mathematical functions, which are interpretable and represent various dynamical characteristics of a physical system [4]. Then, we use this pre-defined library in the purview of sparse Bayesian regression to discover the functional forms of the perturbations. The proposed framework has the following contributions: firstly, the exact mathematical descriptions of the perturbation are discovered, making the updated DTs highly interpretable and generalizable to unseen environmental conditions. Secondly, being Bayesian, the proposed approach also provides model-form uncertainties, which are otherwise missing in the data-driven ML-based DT frameworks.

2. METHODOLOGY

We consider a dynamical system of the form,

$$\dot{\boldsymbol{X}}_t = \boldsymbol{f}(\boldsymbol{X}_t, t) + \boldsymbol{h}(\boldsymbol{X}_t, t); t > 0; \boldsymbol{X}(0) = \boldsymbol{X}_0,$$

where $\boldsymbol{f}(\boldsymbol{X}_t, t)$ and $\boldsymbol{h}(\boldsymbol{X}_t, t)$ represents the system dynamics and perturbation, respectively. Since the initial nominal model $\boldsymbol{f}(\boldsymbol{X}_t, t)$ is known to us a priori, we rephrase the equation as, $\boldsymbol{h}(\boldsymbol{X}_t, t) = \dot{\boldsymbol{X}}_t - \boldsymbol{f}(\boldsymbol{X}_t, t)$. In order to learn the interpretable form for $\boldsymbol{h}(\boldsymbol{X}_t, t)$, we express it as a weighted linear combination of pre-defined interpretable mathematical functions as,

$$\boldsymbol{h}(\boldsymbol{X}_t, t) = \theta_1 + \theta_X \boldsymbol{X} + \theta_{X^2} \boldsymbol{X}^2 + \theta_{XX^2} \boldsymbol{X}\boldsymbol{X}^2 + \ldots + \theta_{\sin(X)} sin(\boldsymbol{X}) + \theta_{\cos(X)} cos(\boldsymbol{X}) + \theta_{|X|} |\boldsymbol{X}| + \theta_{X|X|} \boldsymbol{X}|\boldsymbol{X}|$$

The above equation is represented as $\boldsymbol{h}(\boldsymbol{X}_t, t) = \mathbf{D}\boldsymbol{\theta} + \boldsymbol{\epsilon}$, where $\mathbf{D} \in \mathbb{R}^{N \times P}$ is the library matrix containing all the possible mathematical functions, $\boldsymbol{\theta} \in \mathbb{R}^P$ is the associated model parameters, and $\boldsymbol{\epsilon} \in \mathbb{R}^N$ is the model miss-match error. N and P denote the number of data points and functions in the library $\mathbf{D}$. Our aim is to obtain a sparse weight vector $\boldsymbol{\theta}$. The posterior distribution of the parameters $\boldsymbol{\theta}$ can be obtained from the Bayes rule as,

$$p(\boldsymbol{\theta}|\boldsymbol{h}) \propto p(\boldsymbol{h}|\boldsymbol{\theta})p(\boldsymbol{\theta})$$

where we consider $p(\boldsymbol{h}|\boldsymbol{\theta}) = \mathcal{N}(\mathbf{D}\boldsymbol{\theta}, \sigma_\epsilon^2\mathbf{I}_N)$, and $p(\boldsymbol{\theta}) = p_{slab}(\boldsymbol{\theta}_r)\prod_{k\in P} p_{spike}(\theta_k)$. Here $p_{spike} = \delta_0$ and $p_{slab}(\boldsymbol{\theta}_r) = \mathcal{N}(\mathbf{0}, \sigma_\epsilon^2 v_s \mathbf{I}_r)$ denote the Dirac-delta spike and slab distribution with v_s being the slab variance. Employing a Gibbs sampler, we obtain the posterior distribution of the model parameters $\boldsymbol{\theta}$.

3. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

We consider a single-degree-of-freedom (SDOF) linear mass-spring-dashpot system for numerical illustration, assuming the perturbation as a cubic dissipation term. For synthetic data generation, we use the following governing equation of motions,

$$\underbrace{m\ddot{X}(t) + c\dot{X}(t) + kX(t)}_{\text{Initial model}} + \underbrace{\alpha X^3(t)}_{\text{Perturbation}} = \sigma f(t); t > 0; X(0) = X_0,$$

where $m = 1$kg, $c = 2$Ns/m, $k = 1000$N/m, $\alpha = 10^5$, and $\sigma = 0.5$ are the mass, damping, stiffness, nonlinear spring constant, and source strength, respectively. The external source $f(t)$ is simulated as a zero mean Gaussian white noise. The synthetic responses are obtained using fourth-order Runge-Kutta scheme with a Δt=0.001s for a total duration of 1s. The results are shown in Fig. 1, where we can observe that the proposed framework correctly discovers the perturbation term in its interpretable cubic form along with its actual parameters.

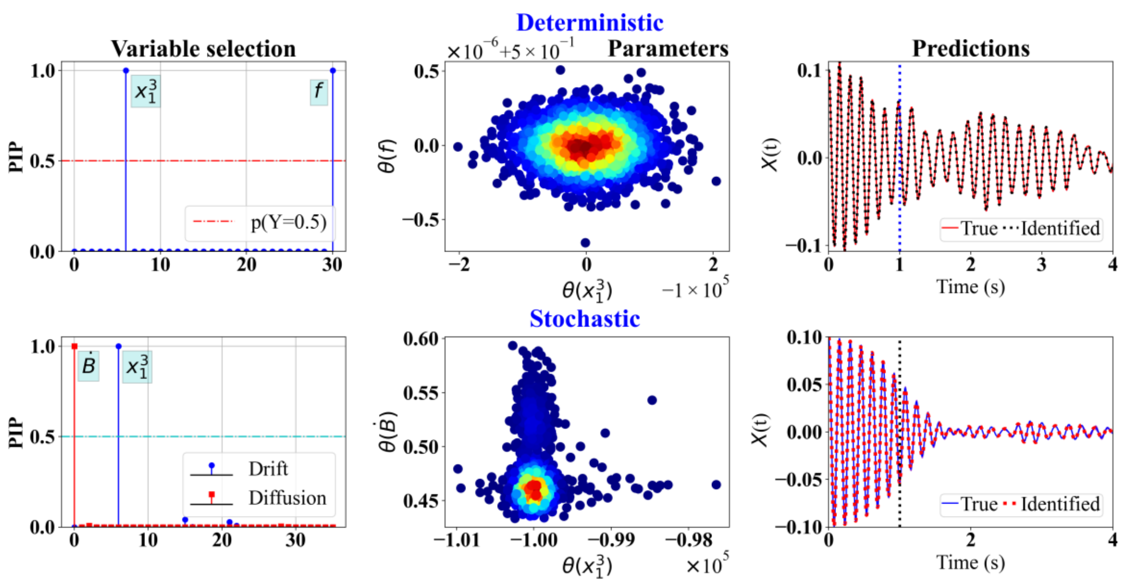

Figure 1. Discovery of the interpretable form of the cubic dissipation term (left), its posterior probability distribution (middle), and prediction using the updated interpretable digital twin model (right).

REFERENCES

1. K. Främling, J. Holmström, T. Ala-Risku, and M. Kärkkäinen, "Product agents for handling information about physical objects," *Report of Laboratory of information processing science series B* **153**, 2003.
2. Q. Liu, B. Liu, G. Wang, and C. Zhang, "A comparative study on digital twin models," *In AIP Conference Proceedings* **2073**, p. 020091, 2019.
3. W. Booyse, D. N. Wilke, and S. Heyns, "Deep digital twins for detection, diagnostics and prognostics," *Mech Syst Signal Process* **140**, p. 106612, 2020.
4. T. Tripura, and S. Chakraborty, "A sparse Bayesian framework for discovering interpretable nonlinear stochastic dynamical systems with Gaussian white noise," *Mech Syst Signal Process* **187**, p. 109939, 2023.