Superconvergent Jacobi Spectral Methods for Nonlinear Volterra-Fredholm Integro-Differential Equations

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1. INTRODUCTION & OBJECTIVE

In this paper, the Jacobi Spectral Galerkin (JSG) and iterated Jacobi Spectral Galerkin (IJSG) methods for a nonlinear Volterra-Fredholm integro-differential equation (NVF-IDE) have been proposed. Jacobi polynomial-based Galerkin and iterated Galerkin techniques have been used to tackle the NVF-IDE. The convergence analysis is carried out for the proposed methods, and error estimates are derived. The IJSG method improves the JSG method in terms of convergence rates. The theoretical outcomes are verified numerically. The proposed scheme will be tested against few test cases and the obtained results will be compared with the state of art methods like FDM/FCM.

(NVF-IDE) are naturally used in a wide range of scientific and technological applications, such as stereology, heat radiation from semi-infinite materials, heat conduction with mixed boundary conditions, crystal formation, superfluidity, gas absorption, and electrochemistry [1],[2],[3].

Consider the following Volterra- Fredholm integro-differential equation

$$\sum_{i=0}^{m} a_i u^{(i)}(x) = f(x) + \lambda_1 \int_0^1 k_1(x,t)[u(t)]^p dt + \lambda_2 \int_0^x k_2(x,t)[u(t)]^q dt, \quad 0 \le x \le 1,$$

Subject to the following conditions

$$u^{(i)}(0) = \beta_i, i = 1,2,\cdots,m$$

where $u(x)$ is the unknown function that has to be found, $f(x)$ is a known function, and $k(x,t)$ is a function of two variables, x and t, known as the kernel. As well as β, λ_1 and λ_2 are constant parameters.

2. RESULTS

This study deals with the application of the Galerkin technique to the Jacobi polynomials in order to solve the NVF-IDE. The findings demonstrate that the Galerkin

technique, which used Jacobi polynomials as basis functions, worked admirably well in terms of the absolute errors achieved. Numerical examples are given for a few test cases. Under the Spectral Galerkin frame work, we have proposed JSG and IJSG methods for NVF-IDE utilising Jacobi polynomials. We also carry out the mathematical convergence analysis for the proposed schemes. Further, we computationally establish the derived order of convergence through numerical test cases. The IJSG method is found to have super convergence trend both on theoretical and computational fronts.

References

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[3] Tahmasbi, Ali, and Omid S. Fard. "Numerical solution of linear Volterra integral equations system of the second kind." Applied Mathematics and Computation 201, no. 1-2 (2008): 547-552.