A Variational multiscale stabilized finite element method for non-Newtonian fluid flow with application to double diffusion process in a partially heated multi-staggered cavity under magnetic field

Dipak Kumar Sahoo [1a], B V Rathish Kumar[1b]

[a]Indian Institute of Technology Kanpur, Address,Kanpur, India
[b] Indian Institute of Technology Kanpur, Address,Kanpur, India

1. INTRODUCTION & OBJECTIVE

Double diffusive natural convection is one of the important problems in the various energy systems including cooling systems, air solar heater, heat exchanger, solar dcryer solar desalination [1, 2]. Different methods have been used to improve the heat transfer rate in various designs. Study of such a intricate convection process in a complex configurations like a multi-staggered cavity under magnetic effect is of great relevance in several engineering and scientific applications. The mathematical models governing such a double diffusive heat-mass transfer under magnetic forces turn out to coupled system of nonlinear partial differential equations. Several research efforts to develop robust numerical methods, which can solve such nonlinear systems in complex geometries under varying force interactions, are being carried out across the world[3]. Here, a novel stabilized finite element method in a subgrid multi-scale variational framework will be proposed to efficiently solve such models in complex geometries. Further a detailed parameteric study will be undertaken to unravel the physics behind such a double-diffusive convective process.

The model equations are given as in non-dimensional form [4]:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$\left(U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)=-\frac{\partial P}{\partial X}+Pr\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)+\Lambda_X \tag{2}$$

$$\left(U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=-\frac{\partial P}{\partial Y}+Pr\left(1+\frac{1}{\beta}\right)\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\Lambda_Y \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

$$U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Le}\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right) \tag{5}$$

Where $\Lambda_x = PrHa^2(V\sin\gamma\cos\gamma - U\sin^2\gamma)$

and $\Lambda_Y = PrHa^2(U\sin\gamma\cos\gamma - V\cos^2\gamma) + RaPr(\theta + NC)$

With boundary conditions:

$U = 0, V = 0, \theta = 1, C = 1$ (Hot side)

$U = 0, V = 0, \theta = 0, C = 0$(Cold side)

$U = 0, V = 0, \frac{\partial \theta}{\partial \boldsymbol{n}} = \frac{\partial C}{\partial \boldsymbol{n}} = 0$ (Remaining walls)

2. NUMERICAL METHOD & RESULTS

A variational multiscale stabilized finite element method for coupled system of nonlinear partial differential equations governing the heat and mass transfer with Boussinesq approximation in a multi-staggered partially heated cavity under magnetic influence will be derived. Apt flow stabilization parameter will be derived using the fourier theory. The obtained stabilized variational problem will be projected into the finite dimensional subspace to be tackled by Newton's qausi-linearization to obtain the approximate solution. The stabilized multiscale finite element scheme will be first validated on the benchmark problems and later will used to trace the physics behind the double diffusive heat and mass transfer in a partially heated multi-staggered cavity system. The influence of flow and heat transfer parameters such as Rayleigh Number, Casson Parameter, Lewis Number, Prandtl Number and Hartmann Number will be analyzed through the streamlines, isotherms and the local heat fluxes to understand the double diffusive natural convective process in the complex cavity.

REFERENCES

[1] Shoeibi, S., Rahbar, N., Esfahlani, A. A., Kargarsharifabad, H. (2021). Energy matrices, exergoeconomic and enviroeconomic analysis of air-cooled and water-cooled solar still: Experimental investigation and numerical simulation. Renewable Energy, 171, 227-244.

[2] Shoeibi, S., Rahbar, N., Esfahlani, A. A., Kargarsharifabad, H. (2020). Application of simultaneous thermoelectric cooling and heating to improve the performance of a solar still: an experimental study and exergy analysis. Applied Energy, 263, 114581.

[3] Computational fluid dynamics by tj chung.

[4] Hussain, S., Shoeibi, S., Armaghani, T. (2021). Impact of magnetic field and entropy generation of Casson fluid on double diffusive natural convection in staggered cavity. International Communications in Heat and Mass Transfer, 127, 105520.