Iterative Numerical Method for Nonlinear Moving Boundary Problem with a Convective Boundary Condition

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INTRODUCTION & OBJECTIVE

Many physical phenomena are typically described in terms of boundary value problems (BVPs), in which certain restrictions on the boundaries of a prescribed domain constrain the solution of differential equations (DEs). There are many important processes during which matter changes states and a boundary is generated between them. This type of problem poses the challenge of determining the boundary position as part of the solution because it is unknown prior. A moving boundary problem (MBP) refers to BVP dependent on time, where the position of the moving boundary (MB) is determined over space and time. Josef Stefan [1] developed a model for glacier formation in the polar regions in 1889.

This was the first generalized study of MBPs. So this type of problem is also known as Stefan problem. Unlike BVPs that start with predetermined initial and boundary conditions, MBPs have the added complexity of requiring two extra conditions on the boundary in motion. One of these conditions is needed to establish the initial position of the moving boundary, while the other must fulfil the partial differential equations (PDEs) that regulate the process within each region.

The MBP is important to science and industry, especially in the oil, glass metal, plastic, freezing of foods, ice production industries, etc. Because these problems involve the interaction between unknown geometric quantities (e.g. part of the domain boundary) and unknown physical quantities (e.g., temperatures inside the domain), they are nonlinear. This article focuses on phase transition processes resulting from an external temperature imposed at the fixed boundary of a homogeneous material. The temperature-dependent boundary condition is a classical simplification in the modelling of such phenomena. In this case, the material is imposed with a specific temperature through the instantaneous transmission of heat. This is physically unfeasible, which is why many authors recommended convective boundary conditions [2, 3] because the difference between the imposed temperature and the material's temperature at the boundary is proportional to the heat transfer at the same boundary. The assumption that thermo-physical properties are constant is another way to simplify phase transition models. It seems reasonable for most phenomena under moderate temperature variations, but it does not happen typically. Several models have attempted to improve this assumption by removing it [4, 5, 6, 7, 8]. Our study of phase transition processes with convective boundary conditions and nonlinear physical property has been influenced by all of the above factors.

This paper proposes a numerical scheme based on iterative KBM via the boundary immobilization method (BIM) for the nonlinear MBP with a convective boundary condition. Section 2 discusses how to formulate the model problem and non-dimensionalise it. Section 3 demonstrates how to immobilize the moving boundary using BIM and discretise the nonlinear continuous problem into a system of linear algebraic equations using iterative KBM. A convergence analysis is presented in section 4 of the proposed numerical scheme. The proposed model problem has the similarity solution when it has constant thermal conductivity, described in [2]. Section 5 compares numerical results with the similarity solution of linear MBP with a convective boundary condition, and section 6 concludes our remarks.

RESULTS & HIGHLIGHTS OF IMPOINTANT POINTS

This paper presents an application of iterative-based KBM for a nonlinear MBP with convective boundary conditions. Stability and consistency have shown that the proposed scheme is convergent. The KBM is second-order accurate both spatially and temporally. Numerical simulations calculate the convergence rate of the presented scheme to validate the theoretical results and their accuracy. By considering the corresponding linear form of the MBP with convective boundary condition, it has been found that KBM solutions agree well with exact solutions. Temperature-dependent thermal conductivity has been shown to accelerate freezing front propagation. Furthermore, we found that the speed of the moving boundary increases with an increase in the Stefan number, Biot number, or both. But increases in Stefan or Biot numbers can cause temperature distribution delays in the solid front.

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