Adaptive Phase Field Simulation of Mixed Mode Crack Propagation in Brittle Material

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1. INTRODUCTION & OBJECTIVE

Phase Field Model (PFM) has emerged as an effective tool for modeling various fracture processes in materials, including initiation, propagation, coalescence, and branching. The popularity of PFM arises from its inherent simplicity and flexibility in multi-field problems [1] and diverse material categories [2]. However, one of its disadvantages is the requirement for fine mesh to resolve small-length scale values, which might cause computational challenges. Local mesh refinement is a solution; however, this requires prior information on the fracture path, which is frequently unavailable. To address this issue, this study looks into an adaptive local mesh refinement when its required. This adaptive refining procedure is based on a threshold value of the phase-field variable, which serves as an error indicator. The spatial discretization adapts dynamically during simulation by utilizing quadtree decomposition and by treating elements with hanging nodes as *n*-sided polygons.

2. PROBLEM DESCRIPTION

PFM is a variational technique based on Griffith's theory of brittle fracture. PFM normalizes the acute crack topology by incorporating a smooth and continuous scalar field known as the phase field (ϕ). The term "phase" describes two stages of the loading process for the material: the intact phase (ϕ = 0) and the completely damaged phase (ϕ = 1). Figure 1 depicts the structure of sharp and PF crack in two dimensions.

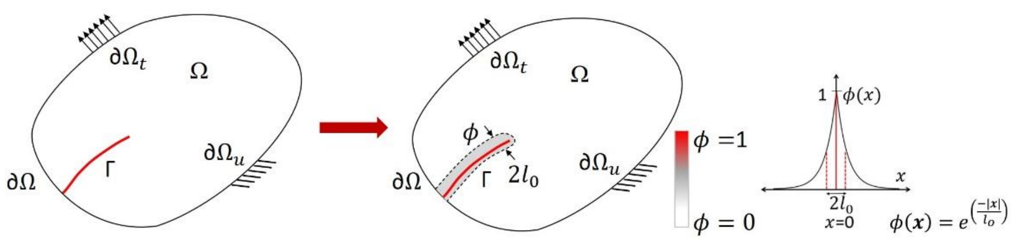


Figure 1. (a) Sharp crack topology and (b) PF crack topology

The governing differential equation for PF fracture simulation is,

Momentum equation: $$[(1-\phi)^2+\beta]\,\nabla.\boldsymbol{\sigma}=0 \quad (1)$$

Damage evolution equation: $$2\phi H^+(\boldsymbol{\epsilon})+\frac{G_c\phi}{l}-G_c l\Delta\phi=2H^+(\boldsymbol{\epsilon}) \quad (2)$$

In the present work, a hybrid phase field formulation is utilized, and the coupled elasticity and phase-field equations are solved using a staggered method [3]. The weak form and the discretized weak form for the evolution equation are developed by using Galerkin finite element formulation. The stiffness matrix and force vector for the damage evolution equation are given as,

$$K_{ij}^{\phi\phi} = \int_{\Omega} \left\{2(1-\beta)H^{+}\left(N_i N_j\right) + \frac{G_c}{l} N_i N_j + G_c l\left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}\right)\right\} dxdy \qquad (3)$$

$$f_i^{\phi} = \int_{\Omega} 2(1-\beta)H^{+} N_i dxdy \qquad (4)$$

The adaptive phase field model's capability is explored by taking a single-edge notch plate subjected to mixed mode loading as shown in Figure 2. The material parameters employed in the investigation are: Lames constant = 121.15 kN/mm^2, Shear modulus = 80.77 kN/mm^2, critical energy release rate = 2.7 N/mm, and length scale parameter = 0.0055 mm. During the simulation, the displacement increment Δu = 1×10^{-4} mm is used for the first 40 steps and u = 1×10^{-5} mm for the remaining time steps.

3. RESULTS & DISCUSSION

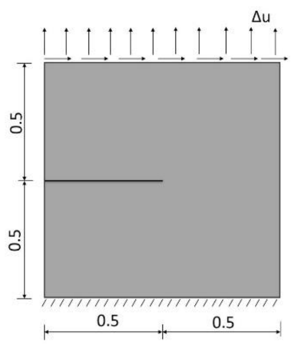
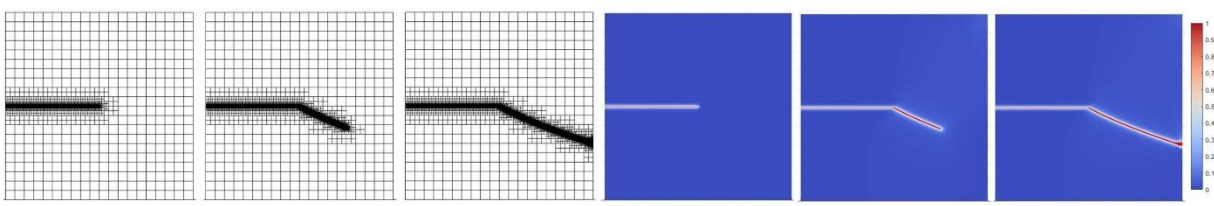


Figure 2. Geometry and boundary condition

Table 1: Comparison of uniform and adaptive mesh refinement

		Uniform Meshing	Adaptive Meshing
Number of Elements	Initial	131044	5176
	Final	131044	12595
Critical Load (in kN)		0.762	0.759
Computational Time (in min)		1564	57



(a) (b) (c) (d) (e) (f)

Figure 3. Single-edge notch plate under mixed mode loading. Adaptive mesh at (a) Δu = 0 mm (b) Δu = 4.86 × 10^{-3} mm (c) Δu = 5.52 × 10^{-3} mm along with the corresponding crack path in (d), (e), and (f), respectively.

Figure 3 depicts the snapshots of the crack path at various displacements to demonstrate how well the adaptive refinement works to maintain local refinement at the fracture tip and how it automatically discretizes as the crack grows. Table 1 shows the computational effectiveness of adaptive mesh refinement. The results reveal that the number of elements is reduced by 90% when compared to uniform mesh, resulting in a 27-times increase in computation efficiency.

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