PAPER FOR THE YOUNG SCIENTIST AWARD

Effect of wave blocking in finite depth two-layer fluid

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1. INTRODUCTION

Most offshore oil and gas reserves are located in deep sea areas requiring pipeline transportation. Such transformation require the pipelines to be entirely underwater (not touching the ocean floor) and subject to the impact of different oceanic parameters and types of waves. Wave propagation occurs along both the surface and the interface. The initial process of such a propagation was explained by Stokes [1]. This concept was later generalized for an ice-covered surface by Schulkes et al. [2]. In addition, ocean current plays a vital role in the analysis of progressive waves. Blocking occurs when the group velocity of a wave train equals the magnitude of the opposing current speed. Barman et al. [3] had shown the impact of blocking on the scattering of flexural-gravity waves by considering a linear crack in a thin ice cover.

Although significant works have been devoted to study the wave blocking, limited emphasis have been focused on understanding the effect of compression, ocean current and porosity parameter in wave blocking in a two layer fluid. An effort has been made to understand the same in a single-layer fluid in the absence of porous bed, and the results are presented in the work of Das et al. [4]. This work aims to extend the study to a two-layer fluid with porous bed, making it more exciting and challenging to handle due to more coupled propagating modes.

MATHEMATICAL FORMULATION

We consider a two-layer fluid of finite depth in which the upper layer is covered by a thin uniform ice sheet that behaves like an elastic plate, and a rigid infinite horizontal bottom bounds the lower layer. Let us consider ρ_I be the constant density of upper layer fluid which occupies the region $0 < z < h$, $-\infty < x < \infty$, $-\infty < y < \infty$ with $z = h$ as the mean position of the thin ice-cover and ρ be the constant density of lower layer fluid occupying the region $d < z < 0$, $-\infty < x < \infty$, $-\infty < y < \infty$ with $z = 0$ as the mean position of the interface and $z = -d$ as the porous bottom with parameter G. The total depth of the fluid is H. Let $\Phi_j(x, y, z, t)$, $j = I, II$ be the complex velocity potentials which describe the fluid motion. The small displacement at the upper surface and interface be considered as $\eta_I(x, y, z, t)$ and $\eta_{II}(x, y, z, t)$, respectively

The velocity potential $\emptyset_j(x, y, z, t)$ satisfy the partial differential equation

$$\frac{\partial^2 \emptyset_j}{\partial x^2} + \frac{\partial^2 \emptyset_j}{\partial y^2} + \frac{\partial^2 \emptyset_j}{\partial z^2} = 0.$$

The linearized kinematic conditions at the mean free surface and interface are

$$\mathcal{D}\eta_I = \frac{\partial \emptyset_I}{\partial z} \quad \text{on} \quad z = h,$$

$$\mathcal{D}\eta_{II} = \frac{\partial \emptyset_I}{\partial z} = \frac{\partial \emptyset_{II}}{\partial z} \quad \text{on} \quad z = 0.$$

Where $\mathcal{D} = \frac{\partial}{\partial t} + U cos\alpha \frac{\partial}{\partial x}\ U sin\alpha \frac{\partial}{\partial y}$ with U is the magnitude of current and α is its direction measured from the positive x-axis.

$$\left(D \frac{\partial^4}{\partial z^4} - Q \frac{\partial^2}{\partial z^2} + 1 + \frac{\varepsilon}{g} \frac{\partial^2}{\partial t^2}\right) \frac{\partial \emptyset_I}{\partial z} + \frac{1}{g} \mathcal{D}^2 \emptyset_I = 0 \quad \text{on} \quad z = h.$$

The interface condition due to the continuity of velocity and pressure are given by

$$\frac{\partial \emptyset_I}{\partial z} = \frac{\partial \emptyset_{II}}{\partial z} \quad \text{on} \quad z = 0,$$

$$\rho\left(\frac{\partial \emptyset_I}{\partial z} + \frac{1}{g} \mathcal{D} \emptyset_{\boldsymbol{I}}\right) = \left(\frac{\partial \emptyset_{II}}{\partial z} + \frac{1}{g} \mathcal{D} \emptyset_{\boldsymbol{II}}\right) \quad \text{on} \quad z = 0.$$

The impermeable bottom boundary condition is

$$\frac{\partial \emptyset_{II}}{\partial z} - G \emptyset_{II} = 0 \quad \text{on} \quad z = -d.$$

METHOD OF SOLUTION

We assume the lower and upper layer's velocity potential for oblique waves respectively are

$$\emptyset_I(x, y, z, t) = \mathrm{Re}[Z_I(k,z) e^{i(ax + by - \omega t)}],$$

$$\emptyset_{II}(x, y, z, t) = \mathrm{Re}[Z_{II}(k,z) e^{i(ax + by - \omega t)}],$$

by applying the above boundary conditions we get the vertical eigenfunctions and then the required dispersion equation which can be written as :

$$K_r^2[(\rho(k + G \coth kd) + (k \coth kd + G) \coth kh] - k K_r[(1-\rho) \coth kh (k + G \coth kd) +$$

$$\Omega(k)(\rho \coth kh (k + G \coth kd)(k \coth kd + G))] + k^2 \Omega(k)(1-\rho)(k + G \coth kd) = 0$$

Where $K_r = \omega_r^2/g$ and $\Omega(k) = Dk^4 - Qk^2 + 1 - \varepsilon k$

For computational purpose following steps are as follows:

1. For wave blocking, calculate $\frac{dw}{dk}$ from the obtained dispersion equation.
2. We get the four branches of the dispersion graph represented as ω_{ij}, where j =+ or - which represent the same or opposite phase .
3. Now we investigate the effect of the current magnitude, ice-compression and porosity parameter on the wave blocking by considering co-current and opposing current.

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