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Analysis of wave scattering at the common interface of piezoelectric media half-spaces under surface/interface elasticity theory

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ABSTRACT

Research aim - The present study investigates the reflection and refraction of SH-wave at the interface of two piezoelectric media in the scope of surface/interface elasticity. The equation of motion and constitutive relations for both the media have been used to derive the expressions for reflection and refraction coefficients for various reflected and transmitted waves. Numerical simulations have been carried out to observe the effects of surface/interface elasticity parameters on the vibrational amplitude ratios. A particular case has been deduced to validate the present study.

Literature survey – In smart structures like SAW devices, when the surface/interface area to the bulk ratio of certain components of interest is large, the produced field quantities are affected by surface/interface electromechanical properties. Thus, surface/interface effects must be considered to determine electromechanical fields accurately. It is generally known that the classical theory of elasticity cannot capture surface and interface effects; hence it cannot predict surface wave propagation. The classical linear theory of elasticity predicts non-dispersive Rayleigh surface wave propagation and no Love wave propagation in a homogeneous half-space [1]. To prevail this weakness of the classical theory, Gurtin and Murdoch proposed the surface/interface elasticity theory [2]. Enzevaee and Shodja [3] have shown that dispersive Rayleigh and Love surface waves may propagate in a homogeneous half-space composed of a fcc crystal. Zhang et al. [4], [5] studied Rayleigh wave and shear horizontal surface waves in a layered piezoelectric nanostructure with surface effects. Some other noteworthy works on reflection/transmission of elastic waves are contributed by Dhua et al. ([6], [7], [8]). But so far, no study has been done regarding reflection/transmission phenomenon of SH-waves in piezoelectric media considering surface/interface effects.

Problem formulation – For the piezoelectric layer with initial stress, the equilibrium equations of elasticity without body forces and the Gauss' law of electrostatics without free charge are given as follows (Pang et al. [9])

$$\sigma_{ij,j} + \left(u_{i,k}\sigma_{kj}^{0}\right)_{,j} = \rho \ddot{u}_i, \tag{1}$$

$$D_{i,i} = 0 \tag{2}$$

where $i, j, k = 1,2,3, \rho$ is the mass density, u_i and D_i denote the mechanical and electric displacements in the i th direction respectively, σ_{ij} is the stress tensor, σ_{kj}^{0} is the initial stress tensor. The dots denote time differentiation, the comma denotes space-coordinate differentiation.

We consider the solution for Equations (1) and (2) as

$$\left(\vec{u}^{(n)}, \phi^{(n)}\right) = (A_n, B_n)\vec{d}_n e^{ik_n(\vec{x}\cdot\vec{p} - c_n t)} \tag{3}$$

where the index n corresponds to respective (reflected/refracted) waves, $\vec{d}$ denote the unit displacement vector, $\vec{p}$ is the unit propagation vector, $\vec{x}$ is the position vector with $\vec{x}\cdot\vec{p} =$ const. A_n and B_n are the vibrational amplitudes, c_n is the velocity of propagation, and k_n is the corresponding wave number.

A shear wave is considered to be propagating in the positive direction of the x_1-axis in order to study the reflection/transmission phenomena of SH-wave in the piezoelectric material. The components of displacement and potential in taken in the form:

$$u_3^n \equiv u_3^n(x_1, x_2, t),\ \phi^n \equiv \phi^n(x_1, x_2, t) \tag{4}$$

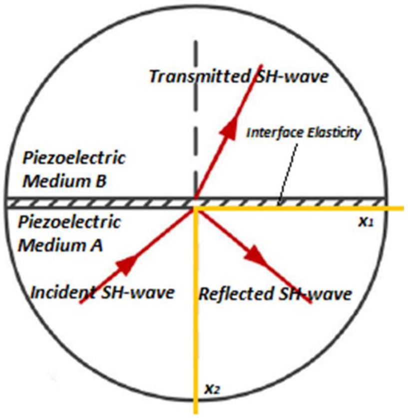

Fig 1. Reflection and the transmission of wave for an incident SH-wave

Boundary Condition – The mathematical framework of surface elasticity in which the surfaces and interfaces are considered as two-dimensional linearly elastic layers of zero thickness. Consider that the interface between the medium *A* and *B* is modeled by the thin membrane. Assuming that the thickness of the membrane is very small compared with the wavelengths of the elastic waves, the displacement and electric potential continuity condition at the interface can then be expressed as

$$u_3^A = u_3^B \tag{5}$$

$$\phi^A = \phi^B \tag{6}$$

Unlike the classical situation, the presence of surface effects induces the jumps of stresses and electric displacements across the surface, which, under the anti-plane deformation, can be described by the nonclassical mechanical and electrical boundary conditions as

$$\sigma_{31,1}^s + (\sigma_{32}^A - \sigma_{32}^B) = \rho^s \ddot{u}_3^s \tag{7}$$

$$D_{1,1}^s = D_2^A - D_2^B \tag{8}$$

Significant conclusions – Size-dependent dispersion properties occurring with the surface effects are predicted, and they may vanish when the film thickness exceeds a critical value. The vibration amplitude ratios are significantly acffected by surface/interface elasticity parameters. the This investigation may have possible applications in signal processing, transduction, frequency shifting (a change in the velocity of surface waves and controlling the selectivity of a filter compensation) of individual devices.

Keywords – Reflection, Refraction, SH-waves, Surface/interfacial elasticity, Piezoelectricity

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