Modeling of Circular Shaft with Inclined Edge Crack for Estimation of Natural Frequencies

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1. ABSTRACT

Cracks can occur in materials due to manufacturing flaws or external loads from adverse service environment. These discontinuities can be present on the surface or beneath it. They may have a longitudinal or transverse orientation, or can also occur on a plane inclined to the longitudinal axis, or on an oblique plane. The work in this paper aims at calculating natural frequencies of a circular shaft with an inclined open edge crack. The crack is defined by its parameters-depth, location and inclination with the longitudinal axis. Strain Energy Release Rate (SERR) approach is used in this work to develop the model of the crack. Using Stress Intensity Factor, compliance coefficients are obtained for a cantilever shaft subjected to the general loading. Stiffness matrix is obtained by inverting the compliance matrix. The forward problem is attempted for calculations of natural frequencies of the Euler-Bernoulli (long) shaft under the influence loads P_1 and P_5 using the relevant stiffness coefficients.

2. INTRODUCTION & LITERATURE SURVEY

Crack in mechanical component provide a serious risk since they can cause financial loss and even put lives in danger. It is therefore essential to find these cracks as soon as possible. Cracks can exhist in the raw materials, during the manufacturing process, or as a result of dynamic loads during operation. Papadopoulos and Dimarogonas [1] have investigated the coupling of longitudinal and bending vibrations in a rotating shaft with an open transverse surface crack. It reveals significant coupling and instability due to the crack providing a basis for crack identification in rotating shafts. Naik and Maiti [2] have introduced a comprehensive model for analyzing free vibration in Timoshenko and Euler-Bernoulli shaft beams with open edge cracks in various orientations. They considered the crack to lie in the transverse plane and used the SERR approach to model the crack. Thakre et al. [3] have estimated natural frequencies for a rectangular long beam in cantilever configuration with an inclined edge crack using linear elastic fracture mechanics. SERR approach is used to compute compliance coefficients and analyze natural frequencies for different crack parameters. In this paper a problem of calculation of natural frequencies of a long circular shaft with an inclined edge crack is attempted.

3. PROCEDURE

Fig. 1 shows a shaft with an inclined edge crack subjected to general loading P_1 to P_6. The loads are transformed on in the new coordinate system aligned with the plane of the crack. Contribution from the loads, in the three fundamental modes of fracture, acting on plane of the crack is considered to obtain SERR for deriving compliance coefficients. The whole cross section, is assumed to be divided into thin longitudinal strips of rectangular cross section of size h by dz, each assumed to be in the state of plane strain.

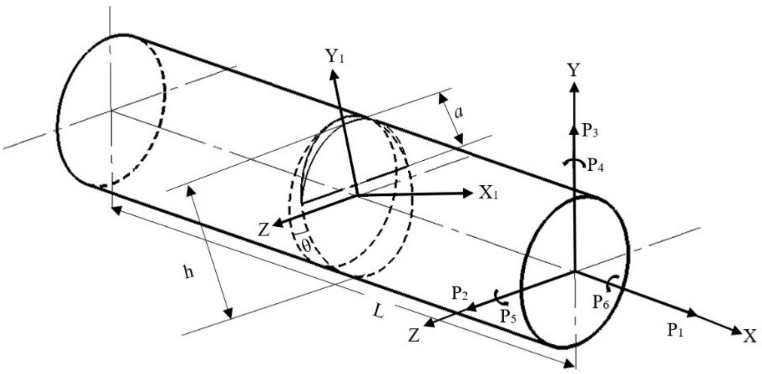

Fig.1 Shaft with inclined crack subjected to general loading

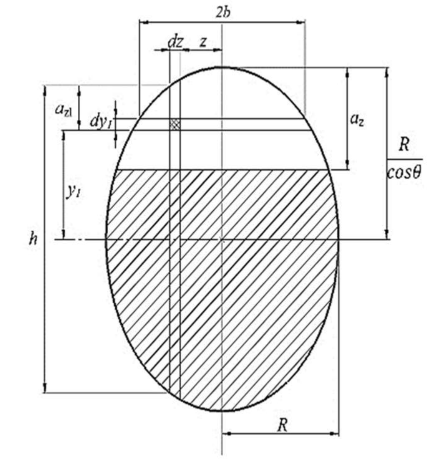

Fig. 2 Elliptical cross-section due to inclined edge crack

For modeling the crack using SERR approach, the additional displacement at the crack location, u_i load in the i^{th} direction is given by the Paris equation:

$$u_i = \frac{\partial U}{\partial P_i} = \frac{\partial}{\partial P_i}\int_{A_c} J\, dA$$

where, U is the energy release rate, A_c is area of the crack and J is the function of the SEER. The compliance coefficients, C_{ij}, are calculated using equation:

$$C_{ij} = \frac{\partial^2}{\partial P_i \partial P_j}\int_{A_c} J(a)\, da$$

Further, to model the coupled transverse and axial vibration in the presence of an inclined edge crack, of the shaft is considered to be subjected to loads P_1 and P_5 only. The natural frequencies for the cantilever shaft are obtained considering it as an Euler-Bernoulli beam. Boundary, continuity and jump conditions are used to arrive at the characteristic equation. The jump conditions need stiffness coefficients which are obtained by inverting the compliance matrix for the loads P_1 and P_5. Natural frequencies are the solution of the characteristic equation for the eigen value problem.

4. RESULTS & HIGHLIGHTS OF THE IMPORTANT POINTS

For comparison of theoretical results, Finite Element Analysis (FEA) is carried out to obtain the natural frequencies of the shaft for different crack parameters values viz. relative crack depth (*a/h*): 0.1, 0.2, 0.3, 0.4 and 0.5; dimensionless crack location, β=0.2, 0.4 and 0.6 and crack inclination, θ=15^0, 30^0, 45^0 and 60^0. Thus, totally 60 cases are considered for computing frequencies using ANSYS Workbench 2022 R1. The same cases may be considered for the experimental work to be carried out in future.

For calculating the natural frequencies for general loading using the crack model developed, a full compliance matrix is obtained. Compliance coefficients essential for determination of natural frequencies under the influence of loads P_1 and P_5 are computed. The theoretical solution for estimation of the natural frequencies is developed and the results are expected to come out soon in the near future. Comparison of the theoretical and numerical results using FEA can be presented in the full-length paper.

5. CONCLUSION

The FE results are studies for now and based on these results the following conclusions are drawn.

(i) As the crack inclination goes on increasing, changes in the natural frequencies due to the crack go on decreasing for a given crack depth and location.
(ii) As the value of crack location increases, changes in the natural frequencies decrease for a given crack inclination and depth.
(iii) As the relative crack depth goes on increasing, changes in the natural frequencies go on increasing for a given crack location in inclination.

The theoretical results are expected to follow the above trend.

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