Effect of thermal modulation parameters on the stability of Rayleigh-Bénard convection in a box

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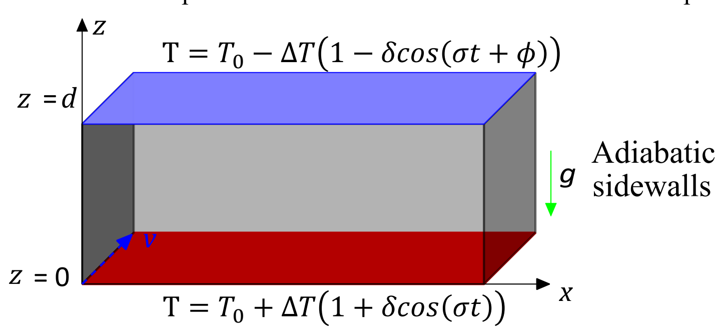
1. INTRODUCTION & OBJECTIVE

We address the generalization of the classical Rayleigh-Bénard convection problem in a horizontal porous layer in an infinitely large domain heated from below to a finite three-dimensional box. We look into a more intricate form of the modulated Rayleigh-Bénard problem in which the temperature at the bottom boundary varies sinusoidally. The critical Rayleigh number is determined using linear and nonlinear stability analyses using the energy method [1]. We observe that modulation amplitude more significantly triggers a change in flow patterns compared to the effect of other parameters. We report the critical temperature difference required for the onset of convection. In addition, a comparison between such temperature differences is also provided. It is observed that subharmonic instability occurs.

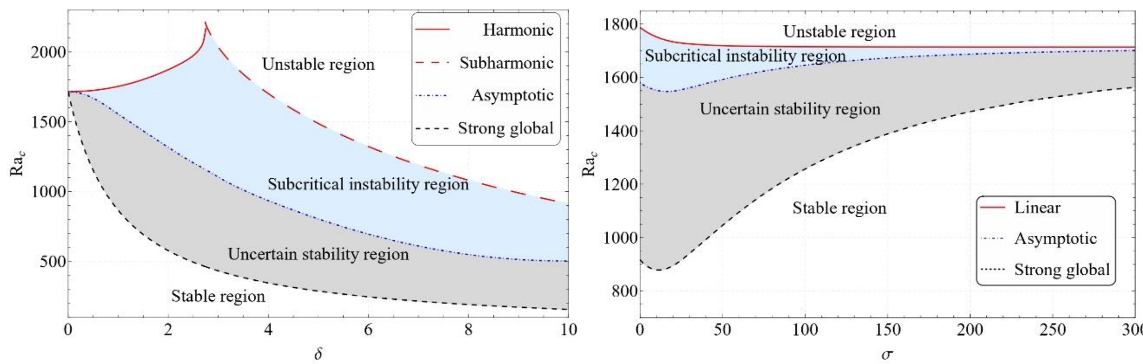
Beck [2] utilized linear theory and the energy method to determine the effect of lateral walls on the onset of convection in a porous medium and observed that both theories provide equal stability bounds. Homsy [3] utilized the energy method for the temperature modulation-driven convection problem and observed that the strong stability limits were lower than the asymptotic ones; also, subcritical instabilities may exist as the energy stability bounds differed from the linear ones. It is apparent from the literature survey that additional work is needed to describe the flow patterns' dependency on temperature modulation and sidewall effects. To the best of authors' knowledge, no study has considered temperature modulation affected flows in a box. Beck [2] obtained the flow patterns for different aspect ratios at the onset of Darcy-Bénard convection. Homsy [3] investigated the stability of time-dependent RBC using the energy method and performed calculations for temperature modulation amplitude (δ) up to 3. The first of the two objectives in the present investigation is to extend the calculations to δ = 10. The present problem, as shown in Fig.1, considers the sidewall effect on the onset of thermally modulated convection. The second objective is to describe the motion in a box resulting from modulation amplitude and frequency variations. The results indicate that modulation amplitude triggers a change in the preferred cellular mode. This objective includes identifying the types of flow patterns and the nature of the transitions among flow patterns.

Figure 1. Schematic of bounded porous domain heated from below with a time-periodic temperature



2. RESULTS & CONCLUSIONS

It is observed that subcritical instabilities are possible in the current investigation, as energy bounds do not coincide with linear ones. Also, similar to Homsy [3], strong global stability bounds are lower than the asymptotic ones as shown in Fig 2. Further, it is observed that with an increase in δ, opposite behavior of the critical Rayleigh number can be observed for linear and energy limits. Linear bound increases with an increase in δ up to a critical value, and sub-harmonic mode becomes the preferred for further increase in δ. In contrast, strong global and asymptotic limits decrease with an increase in δ. The decreasing rate of strong global limits with δ is greater than that of Asymptotic limits. Further, it is observed that the stability limits by linear and nonlinear analyses coincide for $\delta = 0$, indicating that subcritical instabilities occur only due to temperature modulation. This observation is similar to results reported earlier by Beck [2], which show that the energy stability bounds are equal to the linear ones for Rayleigh-Bénard convection.



(a) Ra_c versus δ with $\sigma = 10$,

(b) Ra_c versus σ with $\delta = 1$,

Figure 2: Ra_c obtained for $A_1 = 2.5$, $A_2 = 1.5$, and Pr = 5.

We find that stability bounds obtained using the linear stability and the energy method coincide when $\sigma \rightarrow \infty$, which is a case of non-modulated convection. This observation is analogous to the results reported earlier in various studies [4,5].

We conclude that at low δ values, harmonic instability is identified as the most unstable mode for the considered case. In contrast, the sub-harmonic mode becomes the most unstable one at moderate values of δ. Larger values of σ delay the occurrence of sub-harmonic instability. The critical temperature difference required for the onset of convection is obtained for water using the thermophysical properties. A comparison between the results of linear analysis and the energy method is documented for varying δ and σ, and it is observed that subcritical instabilities as energy bounds are lower than those of linear ones.

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