Linear and Energy Stability of Bénard-Brinkman Convection: Impact of LTNE and Heat Source

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1. INTRODUCTION & OBJECTIVE

The occurrence of penetrative convection is significant across various fields, including geophysics, astrophysics, and fluid dynamics applications. In 1963, Veronis conducted experiments on penetrative convection in a water layer with different temperatures at the upper and lower boundaries, providing a theoretical explanation for the occurrence of finite amplitude instability. There are several methods to induce penetrative convection in a fluid layer, including using a heat source or sink within the fluid layer and a non-linear density-temperature relationship (Normand and Azouni, 1992; Chasnov and Tse, 2001). In systems with substantial velocities, the breakdown of local thermal equilibrium requires using a pair of energy equations for the solid and fluid phases. Banu and Rees (2002) studied the onset of convection in a horizontal layer by incorporating a two-field model of solid and fluid phase temperature fields. The present analysis examines the impact of internal heat generation on Bénard-Brinkman convection in a porous medium characterized by local thermal non-equilibrium between phases. This endeavour involves a thorough nonlinear analysis to assess the potential existence of instability.

Mathematical Formulation: The governing equations and boundary conditions considered are formulated in a dimensionless manner as presented below:

$$\nabla \cdot \mathbf{q} = 0, \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} = -\nabla P - \frac{\mu_f}{K}\mathbf{q} + \mu'_f \nabla^2 \mathbf{q} + \rho_f \alpha T_f \mathbf{g}\hat{k} \tag{2}$$

$$\varepsilon \frac{\partial T_f}{\partial t} + \mathbf{q}\cdot\nabla T_f = \varepsilon\kappa\nabla^2 T_f + \frac{h}{(\rho c)_f}\left(T_s - T_f\right) + Q_f \tag{3}$$

$$(1-\varepsilon)(\rho c)_s \frac{\partial T_s}{\partial t} = (1-\varepsilon)k_s\nabla^2 T_s + h\left(T_s - T_f\right) + Q_s \tag{4}$$

$$\mathbf{q} = \theta = \phi = 0 \qquad \text{at} \qquad z = 0,1. \tag{5}$$

Methodology: The periodic boundary condition for longitudinal rolls is the one to be taken into account when z is involved. The internal-Darcy-Rayleigh-number $\left(Ra_{IDc}\right)$ is obtained via normal mode analysis. The following BEVP is obtained from the analysis:

$$\frac{\sigma}{Va}\left(4D^2 - a^2\right)W = \tilde{D}a\left(4D^2 - a^2\right)^2 W - \left(4D^2 - a^2\right)W - \sqrt{Ra_{ID}}\,a^2\Theta \tag{6}$$

$$\sigma\Theta = \frac{\sqrt{Ra_{ID}}}{\varepsilon} n(z) W + \left(4D^2 - a^2\right)\Theta + H\left(\Phi - \Theta\right) \tag{7}$$

$$\sigma A\Phi = \left(4D^2 - a^2\right)\Phi - \psi H\left(\Phi - \Theta\right) \tag{8}$$

$$W = \Theta = \Phi = 0 \qquad \text{at} \qquad z = -1,1. \tag{9}$$

2. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

The research explores the changes in the critical internal Darcy-Rayleigh number by employing linear analysis $\left(Ra^L_{IDc}\right)$ and the energy method $\left(Ra^E_{IDc}\right)$. This is accomplished through adjustments to the internal heat parameter (ξ), Darcy-Brinkman number $(\tilde{D}a)$, diffusivity ratio (Λ), inter-phase heat transfer parameter (H), and the porosity-modified conductivity ratio (ψ). The findings, illustrated in Table 1, indicate that when the system is heated from above, it may become unstable due to the heat source. However, when heated from below, the system remains comparatively stable. The two analyses demonstrate quantitative discrepancies. Subcritical instability is noticed for each parameter, attributed to internal heat generation. Additionally, alterations in the parameter values mentioned lead to cell size variations, thereby affecting the heat-generating function. An increase in the inter-phase parameter and Darcy-Brinkman number causes a delay in convection, consequently stabilizing the system (Figure 1). Conversely, elevations in the porosity-modified conductivity ratio and internal heat parameter expedite the onset of convection (Figure 1).

Tabel 1: Variation in Critical Internal Darcy-Rayleigh Number by Linear and Non-Linear Analyses for Different Values of Internal Heat Parameter and $\tilde{D}a = 0, \Lambda = 0.5, \psi = 0.1, H = 1.$.

ξ	a_c^L	Ra_{IDc}^L	a_c^E	λ_1	λ_2	Ra_{IDc}^E
0.1	3.92	97.732	3.73	10	0.63	88.217
0.2	3.51	63.072	3.48	10	0.82	59.240
0.3	3.37	45.178	3.37	10	1.04	43.502
0.4	3.31	34.859	3.31	10	1.29	34.023

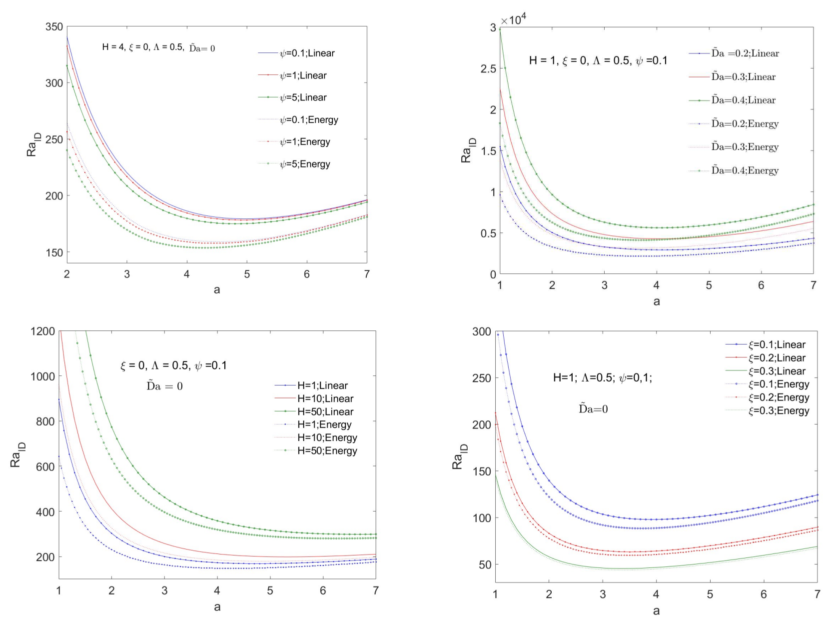

Figure 1 Neutral stationary stability curve for different values of $H, \tilde{D}a, \xi$ and ψ.

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