The Riemann Problem and Interaction of Weak Shock Waves in a Traffic Flow Model with Van der Waals Gas

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Abstract

This investigation addresses the Riemann problem for a traffic flow model involving two velocities in a van der Waals gas. By employing characteristic analysis, we derive the Riemann solution for elementary waves, such as shock waves, rarefaction waves, and contact discontinuity waves. We also provide a detailed explanation of the exact approach regarding a single-parameter family of elementary wave curves. Additionally, we make a general conclusion about the existence and uniqueness of solutions. For initial data involving one and three families of curves, we establish the necessary and sufficient conditions to ensure the presence of shock waves or rarefaction waves. Finally, we delve into a comprehensive study on the interaction of two weak shocks, analysing the intricacies and outcomes of their interaction in depth.

Key words

Riemann problem, two velocities traffic flow model, elementary waves, van der Waals gas, weak shock, wave interactions

Result and Conclusion

The study developed and analyzed a two-velocities traffic flow model using van dar Waals gas. It established explicit solutions for simple, shock waves, and contact discontinuities. The model also presented explicit derivations of elementary wave curves for the Riemann problem. The collision process between two weak shocks was explored. Future research will explore interactions involving strong shock waves and classical elementary waves.

Subsequent inquiries will encompass the exploration of interactions involving strong shock waves and all conceivable wave interactions between classical elementary waves and delta shock waves within the traffic flow model.

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