Solving the Riemann Problem for a Two-Layer Blood Flow Model in Arteries

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1. INTRODUCTION & OBJECTIVE

The Riemann problem typically involves solving a set of hyperbolic partial differential equations with piecewise constant initial conditions. The goal is to find the solution to these equations at a specific point in space and time. This study addresses the Riemann problem in a two-layer blood flow model in arteries, where each layer has the same density but different velocities. The model takes into account the vertical averages across each layer. For instance, due to the viscous effect from the blood vessel, the flow layer near the vessel has a slower average speed than the layer farther from the vessel. We analytically derive the explicit form from the shock, contact discontinuities, and rarefaction waves. We have explicitly derived a family of elementary wave curves that depend on only one parameter. The approach we use, the existence and uniqueness of the Riemann solution are based on the initial data of the solution to the Riemann problem.

2. RESULTS & HIGHLIGHTS OF IMPORTANT POINTS

The Riemann problem for two-layered blood flow model is investigated. We have considered this model with different velocities and equal constant density through arteries. One parameter family of elementary wave curves of solution of the Riemann problem are derived explicitly. The existence and uniqueness of Riemann solution is discussed globally**.**

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