

# Wave Analysis in Porous Thermoelastic Plate with Microtemperature

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## 1. Abstract

The present investigation is focused on wave analysis in porous thermoelastic plates with microtemperature subjected to insulated or isothermal boundaries. After expressing for the two-dimensional case, the governing equations are converted into non-dimensional quantities. Potential functions have been used for further simplification. Normal mode analysis technique is applied to solve the problem. This technique is an influential methodology that furnishes suitable solutions with no presumable limitations. The frequency equations for symmetric and skew-symmetric wave propagation modes are derived from stress-free insulated or isothermal boundaries. The attributes of waves are computed numerically and presented graphically to show the impacts of porosity and microtemperature. Particular cases of the porous thermoelastic plate and the thermoelastic plate with microtemperature are presented. The present problem provides a theoretical foundation for designing and developing new composite materials and surface acoustic devices.

## 2. Research Aim

- a. We want to derive the frequency equations for symmetric and skew-symmetric wave propagation modes for stress-free insulated and isothermal boundaries.
- b. To compute the attributes of waves numerically and graphically to show the impacts of porosity and microtemperature.

## 3. Literature Survey

The linear theory of porous elastic materials is the generalized case of classical elasticity theory. If porosity is negligible, then this theory becomes classical elasticity theory. It is very productive in geological and biological materials, and the linear theory of elasticity needs to be revised. Goodman and Cowin<sup>1</sup> proposed granular theory. Nunziato and Cowin<sup>2</sup> proposed the nonlinear theory of porous elastic material, while Cowin and Nunziato<sup>3</sup> proposed the linear theory of porous elastic materials. Puri and Cowin<sup>4</sup> examined the plane waves in porous thermoelastic material. Iesan<sup>5</sup> extended the linear theory of porous elastic material given by Cowin and Nunziato<sup>3</sup> and included thermal effects. Boldonedo et al.<sup>6</sup> examined the numerical analysis of Lord-Shulman's problem with porosity and microtemperatures.

Grot<sup>7</sup> used Eringen's micromorphic continua theory to develop a thermodynamics theory for elastic materials with microstructure whose microelements possess microdeformation and microtemperature. Riha<sup>8</sup> modified the heat conduction equation involving microtemperature.

Iesan and Quintanilla<sup>9</sup> presented the thermoelasticity theory with microtemperature. Lotfy et al.<sup>10</sup> studied the thermomagnetic effect with microtemperatures in a semiconducting photothermal excitation medium. Aouadi et al.<sup>11</sup> used the GN theory to obtain a non-linear theory of thermoelasticity with microtemperature of type III. They showed that this theory allows the propagation of thermal and microtemperature waves at finite speed with energy dissipation. They also derive governing equations.

#### 4. Problem Formulation

Following Iesan<sup>11</sup>, the governing equations for a homogeneous, isotropic, and porous thermoelastic solid with microtemperature are given as

$$\begin{aligned}(\lambda + \mu)\nabla(\nabla \cdot u_i) + \mu\Delta u_i + b\nabla\phi^* - \beta\nabla T &= \rho\ddot{u}_i, \\ \alpha_1\Delta\phi^* - \mu_1\nabla \cdot w_i - b\nabla \cdot u_i - \xi_1\phi^* + mT - \omega_0\dot{\phi}^* &= \rho\varphi\ddot{\phi}^*, \\ k^*\Delta T + k_1\nabla \cdot w_i - T_0(\beta\nabla \cdot \dot{u}_i + m\dot{\phi}^*) - \rho c^*\dot{T} &= 0, \\ k_6\Delta w_i + (k_4 + k_5)\nabla(\nabla \cdot w_i) - k_3\nabla T + k_2w_i - \mu_1\nabla\phi^* - b_0\dot{w}_i &= 0,\end{aligned}$$

Where  $u_i$ ,  $\phi^*$ ,  $T$ , and  $w_i$  represent displacement, porosity, temperature, and microtemperature, respectively.  $\lambda$  and  $\mu$  are lames constants,  $\rho$  is density, and  $T_0$  is reference temperature.  $\alpha_1$ ,  $\mu_1$ ,  $b$ ,  $\xi_1$ ,  $\omega_0$ ,  $\varphi$  are porosity parameters.  $k^*$  is thermal conductivity, and  $c^*$  is specific heat at constant strain.  $\mu_1$ ,  $b_0$ ,  $k_1$ ,  $k_2$ ,  $k_3$ ,  $k_4$ ,  $k_5$  and  $k_6$  are microtemperature parameters.  $\beta = (3\lambda + 2\mu)\alpha_t$ ,  $\alpha_t$  is thermal expansion.

#### 5. Solution Methodology

Initially, we converted the governing equation into dimensionless form, and then we used the Helmholtz-decomposition technique for simplification. Further, we used Normal Mode Analysis to solve the obtained equations. Also, we have calculated phase velocity and specific loss numerically with the help of MATLAB software and plotted against wavenumber to observe the effect of porosity and microtemperature on these wave attributes.

#### 6. Conclusions

We obtain the frequency equations for symmetric and skew-symmetric wave propagation modes for stress-free insulated and isothermal boundaries after applying boundary conditions. Phase velocity attained a peak for initial values of wavenumber. Further, as the wavenumber increases, phase velocity converges to some value. Specific loss decreases for all values of wavenumber but decreases rapidly for the fundamental mode. For the higher values of wavenumber, specific loss converges.

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