A Study on the Effects of G-gitter and Variable Viscosity and on the Convective Stability of Oldroyd-B Fluids

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ABSTRACT

The effects of variable viscosity and g-gitter is studied on Oldroyd-B fluids undergoing convection of Rayleigh-Bénard type. Both Linear and a nonlinear analysis is done. The eigenvalue characterized by thermal Rayleigh number is found using a perturbation method. An analysis of the linear case shows the stabilising and destabilising effects of strain retardation and stress relaxation parameters. The study of the nonlinear part is performed using expansion of Fourier series. Heat transfer is measured quantitatively using Nusselt number. Variable viscosity is shown to have a destabilising effect. Sub-critical motion is observed due to the imposed modulation.

***Key words*:** Rayleigh-Bénard convection, g-gitter, Gravity modulation, Variable viscosity, and Oldroyd-B liquid.

RESEARCH AIM

This paper aims to investigate the effects of gravity modulation and time-dependent viscosity on Oldroyd-B liquids while undergoing Rayleigh- Bénard convection along with the impact of the parameters on heat transfer.

LITERATURE SURVEY

An in-depth exploration of convection has been around for many years due to its applications in various fields. The above stated is why a vast amount of research has been done on convective stabilities of fluids that are heated below the surface. (Siddheshwar *et. al.* [1], Vest and Apaci [2], Green[3]) The strain to stress parameters' ratio was found to be below 1 in order to set in convection.(Siddheshwar and Krishna [4]) Siddheshwar [5] investigated the stability analysis of viscoelastic liquids undergoing modulation of gravity. His work reinstated the ability to change the frequency of modulation in order to get the desired rate of convection. Sharma[6] found the effects of Coriolis force on Oldroyd-B liquids. Stengel *et. al.* [8] did a study of the convective stability of a variable viscosity fluid. Critical values of the thermal Rayleigh number were found to be the same at small values of the viscosities' ratio. Varying viscosity conductivity of nanofluids on heat transport in convection was studied by EiyadAbu-Nada [9].

MATHEMATICAL FORMULATION

Viscoelastic Oldroyd-B fluid is kept in between parallel surfaces. The coordinated of the two plates are taken to be z = 0 and z = d. These are kept at v degrees of heat. The difference in temperatures is denoted by ΔT. The temperature difference causes a density gradient. The assumption here is that the density of the fluid change linearly with temperature.

Equations (1) – (5) govern the Rayleigh-Benard convection in the liquid under study.

Equation of continuity: $\nabla.\vec{q} = 0$ (1)

Momentum equation: $\rho_0\left(\frac{\partial \vec{q}}{\partial t}\right) = -\nabla p + \rho g_0 (1+\delta\varepsilon\cos\omega t)\vec{k} + \nabla.\tau'$ (2)

Equation of rheology: $\left(1+\lambda_1\frac{\partial}{\partial t}\right)\tau'=\left(\lambda_2\frac{\partial}{\partial t}+1\right)\left(\nabla\vec{q}+\nabla\vec{q}^{tr}\right)\frac{\mu_0}{1+\delta_T(T-T_0)}$ (3)

Conservation of Energy: $(\vec{q}.\nabla)T+\frac{\partial T}{\partial t}=\kappa\nabla^2 T$ (4)

Density Equation: $\rho=\rho_0\left(1-\alpha(T_b-T_0)\right)$ (5)

The non-dimensional parameters appearing in the problem are thermal Rayleigh number, Prandtl number, strain retardation parameter, stress relaxation parameter, and variable viscosity parameter given in equation (6).

$$\left.\Lambda_1=\frac{\lambda_1\kappa}{d^2},\ \Lambda_2=\frac{\lambda_2\kappa}{d^2},\quad \Pr=\frac{\mu}{\rho_0\kappa},\quad Ra=\frac{\alpha\rho_0 g\Delta T d^3}{\mu\kappa}, V=(\delta_T-K_1)\Delta T\right\} \tag{6}$$

Linear Analysis of Convective Stability

Over stability and marginal states are considered in the linear case. Taking only the linear terms into consideration and applying perturbation technique we arrive at the small correction to the thermal Rayleigh number

$$R_{2C}=\frac{1}{2\pi^2\alpha^2k^2\,|L(\Omega,n)|^2}\left[\left(A_2A_4+A_1A_2\Omega+A_3^2\right)Y_1+\left(A_2A_3+A_1A_4\Omega-A_3A_4\right)Y_2\right] \tag{7}$$

where,

$$A_1=-R_0\pi^2\alpha^2-R_0\pi^2\alpha^2k^2\Lambda_1 \tag{8}$$

$$A_2=-R_0\pi^2\alpha^2k^2+R_0\pi^2\alpha^2\Omega^2\Lambda_1 \tag{9}$$

$$A_3=R_0\pi^2\alpha^2\Omega \tag{10}$$

$$A_4=-R_0\pi^2\alpha^2k^2+R_0\pi^2\alpha^2\Omega^2\Lambda_1 \tag{11}$$

Non-Linear Study

The nonlinear study is required in order to understand the extent of heat transport caused by the variations in the parameters. The generalized Lorenz model is given by eq. (12) – (15). The over-dot represents first derivative in t.

$$\dot{A}(t)=\frac{-\Pr R\pi\alpha}{k^2}B(t)-\Lambda G_1(z)k^2\Pr A(t)-\frac{\Pr}{k^2}E(t) \tag{12}$$

$$\dot{B}(t)=-k^2B(t)+(\varepsilon f-1)\pi\alpha A(t) \tag{13}$$

$$\dot{C}(t)=-4\pi^2C(t)+\frac{\pi^2\alpha}{2}B(t)A(t) \tag{14}$$

$$\dot{E}(t)=\frac{(1-\Lambda G_1(z))k^4}{\Lambda_1}A(t)-\frac{1}{\Lambda_1}E(t) \tag{15}$$

RESULTS AND CONCLUSIONS

Graphs are plotted for R_{2c} vs Ω for different values of the parameters. These plots are depicted. One such graph is shown here.

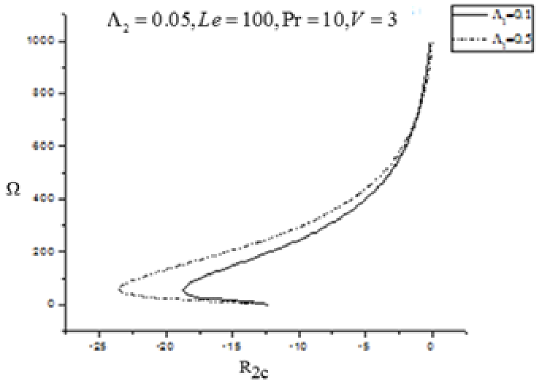

Fig 2: Plot of R_{2c} vs Ω for different values of Λ_1

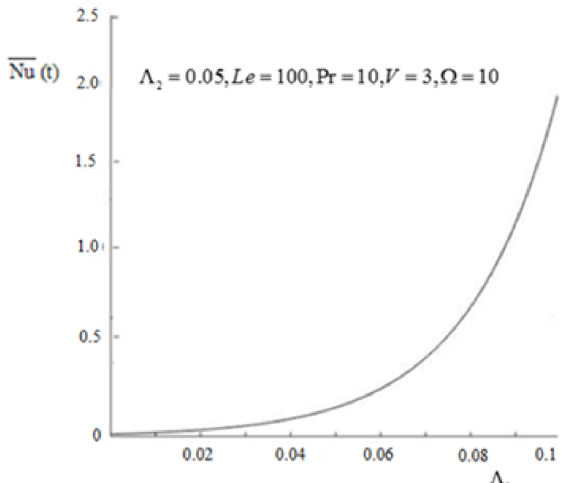

Fig 3: Plot of $\overline{Nu(t)}$ vs Λ_1

Figure (2) is the plot of R_{2c} versus Ω for by changing Λ_1. All other parameters' values stay constant. That is, an increase of in Λ_1 results in a decrease in R_{2c}. This accelerates the convective process. The non-linear graphs are that of average Nusselt number versus the different parameters. The upward trend of the graph in figure (3) shows the increase in average Nusselt number when the values of stress relaxation parameter increase. This reinstates the result observed in figure (2). From other graphs along with the ones shown here we can see the opposing effects are clear in the linear and nonlinear parts of the study. While the stress relaxation parameter, Λ_1 results in more heat transfer, the strain retardation parameter, Λ_2 results in less heat transfer. Gravity modulation frequency decreases the heat transport. Variable viscosity destabilizes the system.

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