A new surface wave dissipation mechanism induced by viscoelastic bed

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1. INTRODUCTION & OBJECTIVE

Wave dissipation refers to the process by which the propagating wave energy through a medium gets reduced. Bottom friction, percolation, turbulence, and bed fluctuations all contribute to wave energy dissipation [1] that are influenced by a variety of factors, including water depth, wave characteristics, and seabed nature. Focusing on the seabed, different types, namely sand-bed, poroelastic bed and mudbed, are often considered [2]. However, muddy seafloors found across most continental shelves cause strong dissipation of ocean surface-gravity waves [3]. Understanding the mechanisms underlying wave dissipation over a muddy seabed involves the evolution of mud rheology, which varies by location. One of the mud rheology models which is quite suitable for approximating a wide range of mud properties is viscoelastic model. Our work proposes a new wave dissipation mechanism, based on the method by Muschietti and Dum [4], and over a viscoelastic bed (Kelvin-Voigt model [5]), in finite water depth.

2. MATHEMATICAL FORMULATION & RESULTS

The mathematical formulation of the problem in 2D cartesian coordinate system (x, z) involving two layers with densities ρ_1 and ρ_2 $(\rho_1 < \rho_2)$ is shown in (Fig. 1). The surface and interface elevation are η_1 and η_2, respectively. The potential functions for both the regions and stream function for viscoelastic region are given by $\Phi_j(x,z,t) = \phi_j(x,z)e^{-i\omega t},\ j = 1, 2$ and $\Psi_2(x,z,t) = \psi_2(x,z)e^{-i\omega t}$ respectively, which satisfies the following BVP –

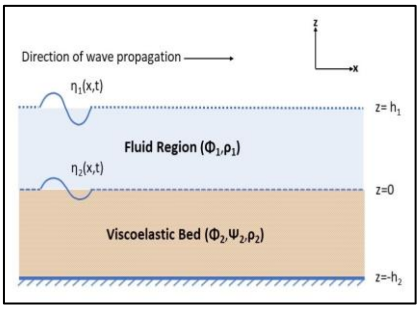

Fig. 1: Schematic Diagram of the Problem

Governing Equations	Boundary Conditions
$\nabla^2\Phi_1 = 0$, in $0 < z < h_1$.	$\frac{\partial^2\Phi_1}{\partial t^2} + g\frac{\partial\Phi_1}{\partial z} = 0$, for $z = h_1$.
$\nabla^2\Phi_2 = 0$, in $-h_2 < z < 0$.	$\frac{\partial\Phi_2}{\partial x} + \frac{\partial\Psi_2}{\partial z} = 0$ & $\frac{\partial\Phi_2}{\partial z} - \frac{\partial\Psi_2}{\partial x} = 0$, for $z = h_2$.
$\nu_e\nabla^2\Psi_2 = \frac{\partial\Psi_2}{\partial t}$, in $-h_2 < z < 0$ with $\nu_e = \nu + i\frac{G}{\omega\rho_2}$, for kinematic viscosity ν & shear modulus of elasticity G.	$\frac{\partial\Phi_1}{\partial z} = \frac{\partial\Phi_2}{\partial z} - \frac{\partial\Psi_2}{\partial x} = \frac{\partial\eta_2}{\partial t}$, $2\frac{\partial^2\Phi_2}{\partial x\partial z} + \frac{\partial^2\Psi_2}{\partial z^2} - \frac{\partial^2\Psi_2}{\partial x^2} = 0$ & $\left(\rho_2\frac{\partial\Phi_2}{\partial t} - \rho_1\frac{\partial\Phi_1}{\partial t}\right) + 2\rho_2\nu_e\left(\frac{\partial^2\Phi_2}{\partial z^2} - \frac{\partial^2\Psi_2}{\partial x\,\partial z}\right) = (\rho_1 - \rho_2)g\eta_2$, for $z = 0$.

Solving the above BVP, will result in the following dispersion relation -

$$\rho_1\left[\frac{(\omega^4-g^2k^2)\tanh kh_1}{gk\tanh kh_1-\omega^2}\right]+2\rho_2k^2\nu_e^2 l\left[\frac{(l^2+k^2)-2k(kC_kC_l-lS_kS_l)}{lS_kC_l-kC_kS_l}\right]-\hat{P}\left[\frac{(l^2+k^2)(lC_kC_l-kS_kS_l)-2k^2l}{lS_kC_l-kC_kS_l}\right]+\rho_2gk=0,$$

where $l=\sqrt{k^2-i\frac{\omega}{\nu_e}}$, $C_k=\cosh kh_2$, $C_l=\cosh lh_2$, $S_k=\sinh kh_2$, $S_l=\sinh lh_2$ and $\hat{P}=\rho_2\nu_e(2k^2\nu_e-i\omega)$. Now, using the method in Muschietti and Dum [4], we study the evolution of a Gaussian wave packet $\mathrm{F}(x,t)$ satisfying the dispersion relation mentioned above. The primary set of equations involved are given below.

$k_{max}(t)=k_c+\Delta^2\Im\left(\frac{d\omega}{dk}\|_{k_{max}}\right)t$	$x_{max}(t)=\Re\left(\frac{d\omega}{dk}\|_{k_{max}}\right)t$
$U_g(t)=\Re\left(\frac{d\omega}{dk}\|_{k_{max}}\right)-\Im\left(\frac{d\omega}{dk}\|_{k_{max}}\right)\frac{\Im(\Lambda)}{\Re(\Lambda)},$	
$\mathrm{F}(x,t)\approx\frac{1}{2\pi}\sqrt{\frac{\Lambda}{\Delta^2}}\exp\left(-\frac{\Lambda(x-x_{max})^2}{2}+ik_{max}x-i\Re(\omega(k_{max}))t+\int_0^t\Im(\omega(k_{max}))dt\right)$	

where k_c is the initial central wave number, Δ is the width of the Gaussian wave packet and $\Lambda(t)=\frac{\Delta^2}{1+i\Delta^2t\left(\frac{d^2\omega}{dk^2}\|_{k_{max}}\right)}$ represents the complex width of the wave packet.

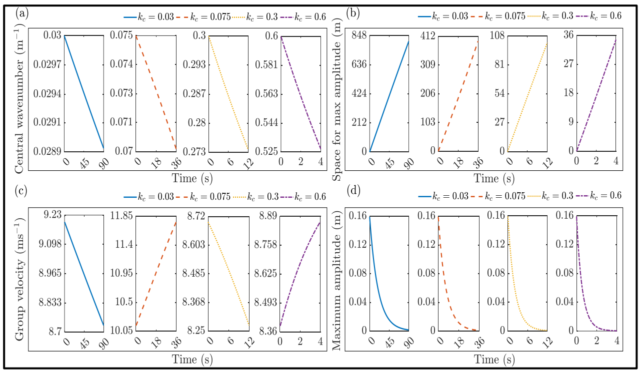

Fig. 2: The transformation of the central wavenumber, group velocities, position and decay of maximum amplitude of Gaussian wave packet are plotted against time.

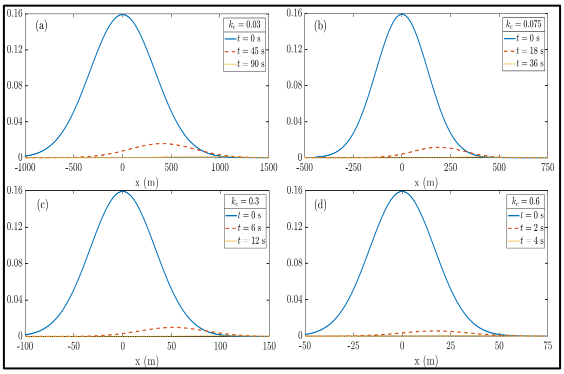

Fig. 3: The evolution of the Gaussian wave packet at three different times for the same initial central wavenumbers in Figure 2 is shown.

In Fig. 2, decreasing group velocity along with the decay of maximum amplitude shows wave dissipation, whereas increasing group velocity along with the decay of maximum amplitude indicates a transfer of energy from kinetic to potential energy. Fig. 3 shows the evolution of Gaussian pulse corresponding to Fig. 2. Additional results and figures will be discussed later.

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