INTERACTION OF GRAVITY WAVES WITH FLOATING MEMBRANE IN PRESENCE OF THICK POROUS BED

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INTRODUCTION

Wave interaction with floating structures offers advantages such as accurately idealising the fluid-structure interaction system and rigorous investigation of wave dynamics. Membranes has the advantageous characteristic of providing coverage over extensive surfaces while maintaining a lightweight composition, low cost, easy manageability, and the ability to be used several times. Consequently, throughout the last decade, there has been a growing fascination with using floating membranes as wave barriers and breakwaters to protect coastal areas and building sites, as well as oil booms and slit curtains to serve unique objectives [1]. Guo et al [2] examined the oblique wave interaction with a submerged horizontal flexible porous membrane in water of finite depth. Yip et al [3] took into account three distinct kinds of edge conditions: fixed, free, and simply supported. They investigated the impact of these edge circumstances on various hydrodynamic coefficients using the eigenfunction expansion approach. Eventually, Karmakar and Sahoo [4] expanded on this research to study surface wave scattering by a semi-infinite horizontal membrane while considering the fixed edge condition. Cho and Kim [5] studied the wave radiation by a horizontal submerged circular membrane. It was found that various types of wave focusing are possible by controlling the membrane parameters. Subsequently, they expanded their research to incorporate the porosity effect of the flexible membrane [6]. The interaction between water waves and a thick porous substrate with a membrane overlay is a very intricate dynamic system that exhibits fascinating behaviours. Although significant work has been devoted to studying the porous structures as breakwaters or barriers, there has been little exploration of using these structures to replicate a thick ocean bed. An effort has been made to understand the same in the presence of a finite floating membrane, making it more challenging and exciting to handle due to the presence of a thick porous bed which is considered significant in this work.

MATHEMATICAL FORMULATION

We consider the small amplitude, linear water wave theory, and the assumption of a finite floating membrane. An inviscid, homogeneous, irrotational and incompressible fluid is considered here. The geometry is sketched using a right-handed coordinate system where $z = -h$ represents the fluid-porous layer interface, $z = -H$ is the impermeable bottom, and $z = 0$ is the free surface. The whole fluid domain is inherited with density ρ. A floating membrane of finite length L is assumed to float on the mean free surface of the water at a finite depth. We examine the waves at the surface that interact with the membrane at an angle θ with the positive direction of the x-axis. The problem is divided into two parts: diffraction and radiation.

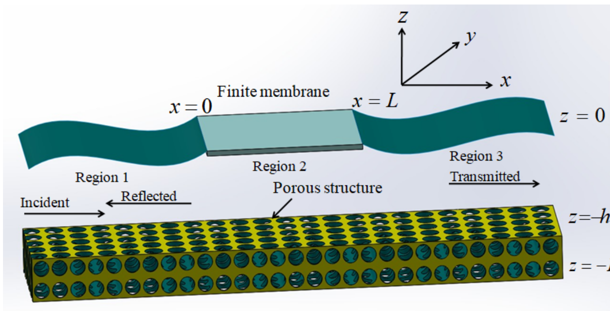

Figure 1. Three-dimensional illustration of water wave interaction with a finite membrane floating over a thick porous bed

For the diffraction and radiation problem

We assume the diffracted potential is in the form of $\Psi_D(x,y,z) = \psi(x,z)e^{\mathrm{i}p_y y}$. the radiated potential is also in the form of $\Psi_R(x,y,z) = -\mathrm{i}\omega\mathcal{R}_a\phi(x,z)e^{\mathrm{i}p_y y}$ where $\mathcal{R}_a$ denotes the amplitude of membrane motion.

$$\psi(x,z) \in \Omega = \begin{cases} \psi^u(x,z), & (x,z) \in \Omega^u, \\ \psi^l(x,z), & (x,z) \in \Omega^l, \end{cases}$$

where $\Omega = \begin{cases} \Omega^u = \{(x,z) : x \in \mathbb{R},\ z \in (-h,0)\}\,, \\ \Omega^l = \{(x,z) : x \in \mathbb{R},\ z \in (-H,-h)\}\,. \end{cases}$ The governing equation is

$$\left(\nabla^2_{x,z} - p_y^2\right)\psi = 0, \quad (x,z) \in \Omega, \tag{1}$$

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the linearized free surface boundary condition yields as follows:

$$\frac{\partial \psi}{\partial z} - \frac{\omega^2}{g}\psi = 0, \quad \text{on} \quad z = 0, \ \forall x \in \{(-\infty, 0) \cup (L, \infty)\}. \tag{2}$$

the linearized condition on the membrane-covered surface is given by on $z = 0, \ \forall x \in (0, L)$,

$$\left[-\frac{\tau}{\rho g}\left(\frac{\partial^2}{\partial x^2} - p_y^2\right) + 1 - \delta\frac{\omega^2}{g}\right]\frac{\partial \psi}{\partial z} - \frac{\omega^2}{g}\psi = \begin{cases} 0, & \text{for diffraction} \\ 1, & \text{for radiation} \end{cases}$$

Equations across the interface are as follows

$$\left.\begin{array}{l} \phi(x, z_-) = (s + \mathrm{i}f)\,\phi(x, z_+), \\ \dfrac{\partial \phi}{\partial z}(x, z_-) = G\dfrac{\partial \phi}{\partial z}(x, z_+), \end{array}\right\} \quad \text{on} \quad z = -h, \ \forall x \in \mathbb{R}. \tag{3}$$

The bottom boundary condition arises as

$$\frac{\partial \phi}{\partial z} = 0, \quad \text{on} \quad z = -H, \ \forall x \in \mathbb{R}. \tag{4}$$

METHOD OF SOLUTION

By using the eigenfunction expansion approach, the velocity potential $\psi_i(x, z)$ in each subdomain $i = 1, 2, 3$ can be written as:

$$\psi_1 = e^{\mathrm{i}q_0 x} Z_0(p_0, z) + \sum_{n=0}^{\infty} R_n e^{-\mathrm{i}q_n x} Z_n(p_n, z), \tag{5a}$$

$$\psi_2 = \sum_{n=0}^{\infty}\left(A_n e^{\mathrm{i}\hat{q}_n x} + B_n e^{-\mathrm{i}\hat{q}_n (x-L)}\right)\hat{Z}_n(k_n, z), \tag{5b}$$

$$\psi_3 = \sum_{n=0}^{\infty} T_n e^{\mathrm{i}q_n (x-L)} Z_n(p_n, z), \tag{5c}$$

where R_n and T_n are the reflection and transmission coefficients of n-th mode.
The following steps has to be followed:

- By using the eigenfunction expansion approach, the velocity potential $\Psi_i(x, z)$ in each subdomain $i = 1, 2, 3$ has to be calculated. Then with the help of above boundary conditions, find a system of linear equations in the form $\mathbb{A}X = \mathbb{B}$.
- From, the obtained system of equations, calculate reflection and transmission coefficients, membrane deflection and vertical force for the diffraction problem, and added mass and damping coefficient for the radiation problem.
- Investigate the effect of frequency, membrane length, membrane tension and porosity on the discussed coefficients.

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