Resonating interaction of flexural gravity waves over sinusoidal seabed

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1 INTRODUCTION & OBJECTIVE

The increasing impacts of global warming and rising sea levels have led to interest in the interaction between gravity waves and floating ice sheets in Polar Regions. This field of study is analogous to surface gravity wave interaction with very large floating structures (VLFSs) since the ice sheets and VLFSs are modeled as thin elastic plates, as in Squire [4]. Examining the various hydrodynamic performances of these floating structures is crucial due to their extensive use in ocean space utilization for humanitarian and technical purposes. While many studies have focused on the reflection of short incident surface gravity waves by man-made structures on the seabed for engineering applications, there is a relative paucity of research on flexural gravity wave scattering by naturally occurring obstacles such as sand ripples and bars. The ability of an undulating seabed to reflect incoming surface gravity wave energy is of considerable interest for seabed ripples. Davies [2] studied the Bragg resonance, which is limited to weak reflection and fails at resonance. When the incident progressive waves interact with the undulated sandbars, the resonating interaction leads to the formation of triads. These conditions can be avoided by introducing a detuning frequency as in Mei[3]. Subsequently, the roles of the detuning frequency on sinusoidal submerged beds of finite lengths is examined. The study of Mei[3] reveals the existence of a cut-off frequency below, in which wave reflection is monotonic in nature, and above, in which it is oscillatory. Couston[1] generalized the study of Mei[3] to a pair of patches and observed the occurrence of Fabry-Perot resonance, which is analogous to the resonance that occurred in wave optics. In this work, using the multiple-scale method, the envelope equation for flexural gravity waves is derived, which is a generalisation of the theory used for surface gravity waves. Subsequently, the role of detuning frequency on flexural gravity wave scattering is studied. We will analyse the scattering coefficients involving the seabed topography in the context of flexural gravity waves for a range of hydrodynamic and ice sheet parameters. Through time-domain simulations, we will demonstrate the intricate phenomenon of wave propagation over sandbars, providing a visual representation of these complex interactions. However, depending on the characteristics of the undulated seabed and the incoming waves, the interaction can amplify or attenuate wave energy. The study will enhance understanding of flexural gravity wave propagation over sinusoidal beds.

2 RESULTS AND HIGHLIGHTS OF IMPORTANT POINTS

Using multiple scale method as in Mei[3], the evolution equation for solving the scattering coefficients in terms of A and B is given by:

$$\frac{\partial A}{\partial t} + c_g \frac{\partial A}{\partial x} = -i\Omega_0 B, \quad \frac{\partial B}{\partial t} - c_g \frac{\partial B}{\partial x} = -i\Omega_0 A. \tag{1}$$

where c_g is the group velocity is given by

$$c_g = \frac{c}{2} \frac{Dk^4 - Qk^2 + 1 - \gamma\omega^2}{Dk^4 - Qk^2 + 1} \left(\frac{5Dk^4 - 3Qk^2 + 1 - \gamma\omega^2}{Dk^4 - Qk^2 + 1 - \gamma\omega^2} + \frac{2kh}{\sinh 2kh} \right),$$

with c being the phase velocity involving flexural gravity waves. Moreover, Ω_0 is the cut-off frequency and is given by

$$\Omega_0 = \frac{Dk^4 - Qk^2 + 1 - \gamma\omega^2}{Dk^4 - Qk^2 + 1} \frac{\omega k \mathcal{D}}{2\sinh 2kh}.$$

Let the wavetrain be slightly detuned due to Bragg resonance with wavenumber $k + \epsilon K$ where K is of order unity. The detuning implies a frequency deviation by the amount $\epsilon\Omega$, where

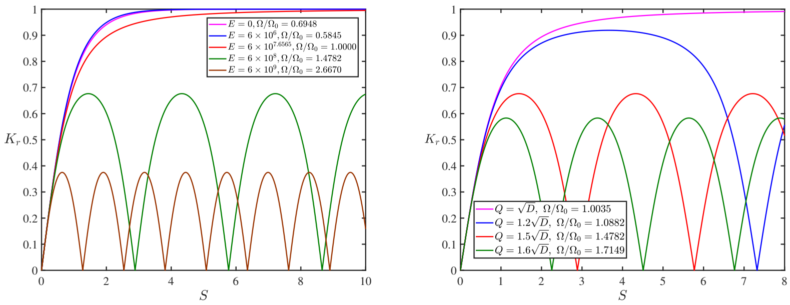
$$\Omega = c_g K$$

The incident wave potential is given by:

$$A = A_0 e^{i(Kx - \Omega t)} = A_0 T(x) e^{-i\Omega t} \quad (x < 0) \tag{2}$$

The differential equations for $x < 0$ and $x > L$ are

$$\left(\frac{\partial}{\partial t} + c_g \frac{\partial}{\partial x} \right) A = 0 \quad (x < 0, x > L) \quad \left(\frac{\partial}{\partial t} - c_g \frac{\partial}{\partial x} \right) B = 0 \quad (x < 0) \tag{3}$$



(a) (b)

Figure 1: Variation of the reflection coefficient K_r versus patch length $S(= \Omega_0 L/c_g)$ for different values of (a) Young's modulus E and (b) compressive force Q with $T = 10$ sec, $h = 5$ m, $K = 0.003$, $d = 1$ m and $D = 1$ m.

As there is no wave arriving from $x = \infty$. Thus,

$$B = 0 \quad (x > L) \tag{4}$$

Case (ii): When the detuning frequency is greater than the cutoff frequency $\Omega > \Omega_0$. Consider the envelope wavenumber P is given by

$$Pc_g = \sqrt{(\Omega^2 - \Omega_0^2)} \tag{5}$$

it can be easily derived that

$$K_r = |R(0)| = \frac{-i\Omega_0 \sin PL}{Pc_g \cos PL - i\Omega \sin PL} \quad T(L) = \frac{Pc_g}{Pc_g \cos PL - i\Omega \sin PL} \tag{6}$$

Similarly, when the detuning frequency is below the cutoff frequency ($\Omega < \Omega_0$). Let

$$Qc_g = \sqrt{(\Omega_0^2 - \Omega^2)}$$

$$T(L) = \frac{iQc_g}{iQc_g \cosh QL + \Omega \sin PL} \tag{7}$$

and

$$K_r = R(0) = \frac{\Omega_0 \sinh Q(L)}{iQc_g \cosh QL + \Omega \sinh QL} \tag{8}$$

. Figure 1 demonstrates the reflection coefficient K_r against $S = \Omega_0 L/c_g$ for flexural gravity waves with $d = 1$. In Fig. 1a, the reflection coefficient against S is plotted for different values of Young's modulus with compressive force $Q = 1.5\sqrt{D}$. As the value of Young's modulus increases, the value of Ω/Ω_0 also increases; hence, the graph pattern changes from monotonicity to oscillatory. Similarly, Fig. 1b depicts the reflection coefficient against S for different values of compressive force Q with Young's modulus $E = 6 \times 10^8$. It is observed that, as the compression increases, the value of Ω/Ω_0 increases and is greater than 1.

References

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