Thermocapillary Migration of a Droplet with Resistive Medium

Rupali Sharma[a], Arindam Basak[a] and G P Raja Sekhar[a]

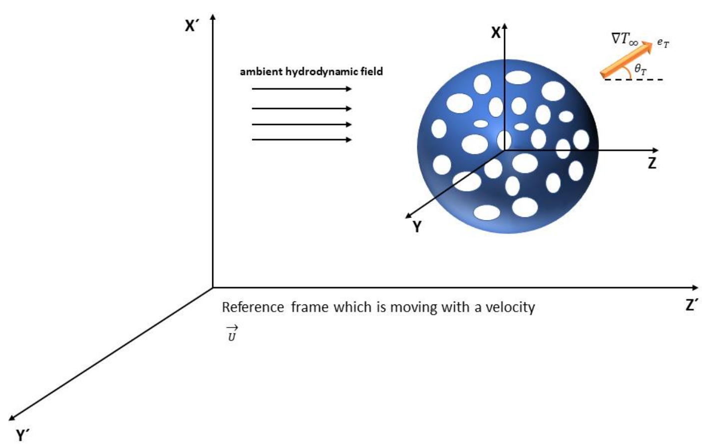
[a] Department of Mathematics, Indian Institute of Technology Kharagpur, West Bengal-721302, India

1. INTRODUCTION & OBJECTIVE

Drug delivery systems (DDS) are essential for enhancing the therapeutic effectiveness of medications by overcoming the challenges associated with conventional drug formulations. Controlled drug delivery systems, in particular, present a promising approach by allowing for sustained drug release over an extended period. Among these systems, hydrogels have garnered significant interest due to their potential in drug delivery applications. Hydrogels consist of a crosslinked three-dimensional polymeric network with a porous structure, and numerous hydrogels have been recently developed for prospective use in drug delivery [1].

In this study, we model the migration of a droplet with an internal scaffold structure subjected to Poiseuille flow under the influence of an external temperature gradient along any arbitrary direction. This temperature variation induces Marangoni stresses along the interface of the porous droplet. We present a novel approach for analyzing the behavior of porous droplet under varying temperature conditions and Darcy numbers, which has the potential to greatly enhance drug delivery systems and contribute to further advancements in the biomedical field.

2. MATHEMATICAL MODELLING



Consider a porous droplet of radius a is suspended in a Poiseuille flow. The flow inside the porous region is governed by the Brinkman equation, with the effective viscosity different from that of the internal fluid viscosity. The linear temperature field T_∞ in the far field fluid medium, given as gradient ∇T_∞=G$\boldsymbol{e}_T$ where, $e_T = \sin\theta_T e_x + \cos\theta_T e_z$. Assuming negligible fluid inertia and thermal convection, the non-dimensional mass, momentum, and energy equation follows,

$$\nabla\cdot\boldsymbol{u}^i = 0, \quad -\epsilon\nabla p^i + (\Delta - \epsilon\lambda^2)\boldsymbol{u}^i = 0, \quad \Delta T^i = 0, \quad \text{for } r < 1,$$

$$\nabla\cdot\boldsymbol{u}^e = 0, \quad -\nabla p^e + \mu\Delta\boldsymbol{u}^e = 0, \quad \Delta T^e = 0, \quad \text{for } r > 1.$$

Here, $\mu = \frac{\mu^e}{\mu^i}$, $\epsilon = \frac{\mu^i}{\mu_{eff}}$ and $\lambda = \frac{1}{\sqrt{Da}}$, $Da = \frac{k}{a^2}$ Da is the Darcy number, k is the permeability and μ_{eff} is the effective viscosity of porous droplet [3]. The kinematic, stress jump, and thermal interface conditions ($r = 1$) are as follows:

1. $\boldsymbol{u}^e \cdot \boldsymbol{n} = 0 = \boldsymbol{u}^i \cdot \boldsymbol{n},$
2. $\boldsymbol{u}^e \cdot \boldsymbol{t} = \boldsymbol{u}^i \cdot \boldsymbol{t},$
3. $\mu \cdot \boldsymbol{\tau}^e \cdot \boldsymbol{n} \cdot \boldsymbol{t} - \frac{1}{\epsilon} \cdot \boldsymbol{\tau}^i \cdot \boldsymbol{n} \cdot \boldsymbol{t} = -\mu \cdot \nabla_s(\mathrm{Ma}_T\, T^e) \cdot \boldsymbol{t},$
4. $T^e = T^i,$
5. $\nabla T^e \cdot \boldsymbol{n} = \kappa\, \nabla T^i \cdot \boldsymbol{n},$

where Ma_T is the thermal Marangoni number, and $\kappa = \frac{\kappa^i}{\kappa^e}$ is the ratio of thermal conductivities.

We now solve the problem within the reference frame moving with velocity $\boldsymbol{U}$. The porous fluid sphere appears stationary in this frame [5]. $\boldsymbol{U}$ is the unknown migration velocity of the porous droplet. Also, far from the porous droplet as $r \to \infty, \boldsymbol{u}^e \to \boldsymbol{v}^\infty - \boldsymbol{U}, p^e \to p^\infty, T^e \to T^\infty$, and internal to the droplet as $r \to 0, \boldsymbol{u}^i, p^i$, and T^i are bounded. $\boldsymbol{v}^\infty, p^\infty$ and T^∞ are due to the applied far-field Poiseuille flow and constant thermal gradient conditions.

We first solve the thermal energy equations considering the interface and boundary conditions. The solenoidal decomposition approach is employed to solve the Brinkman and Stokes equations. Detailed procedures for determining the drag and the resulting migration velocity of the droplet are outlined in Basak et al. [2] and Prakash et al. [4].

3. RESULTS & HIGHLIGHTS

Our results indicate that the presence of an external temperature gradient causes the cross-stream motion of the porous droplet. The thermal Marangoni number accelerates migration. We analyzed the trade-off between the porosity, permeability, effective viscosity parameter, and Marangoni number, identifying zones where migration velocity is optimized. Additionally, the droplet migration velocity for various Da numbers are shown. It is observed that for large Da, droplet migration velocity matches with clear droplet limit [2]. For small Da, the migration velocity agrees with the results of the rigid sphere in stokes flow. This study highlights the significance of thermal effects on the motion of porous droplet and their potential impact on control applications. Our analytical findings provide crucial benchmarks for future computational studies in porous droplet dynamics.

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