# Role of a thin porous vertical barrier and a step-type sea-bed in reflecting waves and mitigating wave forces acting on a submerged tunnel

Nabanita Karmakar[1a] and Swaroop Nandan Bora[1b]

Department of Mathematics, Indian Institute of Technology Guwahati, Guwahati 781039, Assam

[a]Email address:- k.nabanita@iitg.ac.in

[b]Email address:- swaroop@iitg.ac.in

## 1. INTRODUCTION & OBJECTIVE

This problem deals with the oblique water wave interaction with a submerged tunnel in the presence of a thin vertical porous barrier with the sea-bed assumed as an impermeable step-like bottom, and the barrier is placed before the tunnel on the topmost step of the sea-bed. The impact of different parameters, the depth of the sea-bed, the depth of the tunnel, the tunnel width, the distance of the tunnel from the barrier, and the non-dimensional porous-effect parameter of the barrier on the reflection coefficient, the transmission coefficient and the hydrodynamic forces are also carried out.

Underwater tunnels [1] serve a crucial purpose in the modern world by bridging the gap between the land masses separated by vast bodies of water, thereby revolutionizing transportation and connectivity which provide the convenience of traveling from one island to another or crossing a bay without the need of ferries or bridges. Water waves continuously hit the structure and to reduce the waveload on various floating or submerged marine structures, the concept of the use of porous breakwater [2] has been extensively utilized.

We consider the irrotational motion of an inviscid and incompressible fluid over a sea-bed. We use three-dimensional Cartesian coordinate system with the $xy$-plane being the horizontal plane and the $z$-axis pointing upwards and $z = 0$ the undisturbed free surface as in Fig. 1. The barrier, placed at $x=M_1$ with draft h, is assumed to be homogeneous and isotropic in nature. The floating tunnel under consideration is a rigid rectangular structure of finite width $L_3$, draft $d_2$, and at a depth $d_1$ from the free surface. Under the assumptions of linear water wave theory, velocity potential for oblique water wave in each region is given by $\Phi_j(x, y, z, t) = Re\{\varphi_j(x, z)e^{i(ly-\omega t)}\}$, for j = 1,2,…,7 where $\omega$ is angular frequency, $l = k_0 \sin\theta$, $k_0$ is the wavenumber of propagating wave in region 1 and θ is the incident angle with the $x$-axis. The spatial velocity potential $\phi_j$ in each fluid region satisfies the modified Helmholtz equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - l^2\right) \varphi_j = 0, \text{ for j = 1, 2, …,7.} \quad \text{… (1)}$$

The free surface boundary conditions for regions 1, 2, …, 5, and 7 are given by

$$\frac{\partial \varphi_j}{\partial z} - K\varphi_j = 0, \text{ at z=0,} \quad \text{… (2)}$$

where $K = \frac{\omega^2}{g}$, $g$ is the gravitational constant.

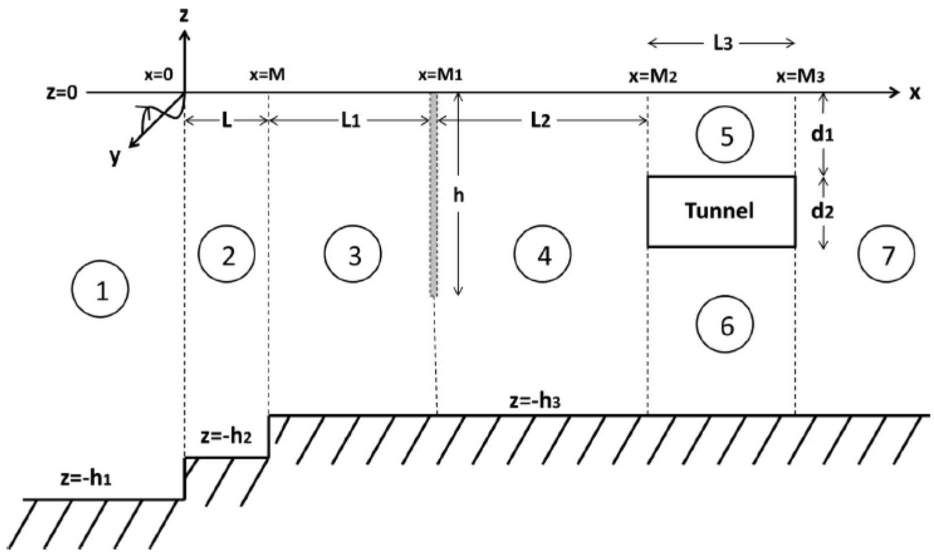



Figure 1: Schematic diagram of the problem

Bottom boundary condition for the impermeable bottom

$\frac{\partial \phi_j}{\partial z} = 0$ at z = $-h_1$ for j = 1, z = $-h_2$ for j = 2, z = $-h_3$, for j = 3,4,6,7. … (3)

Zero horizontal velocity on the vertical sides of the tunnel can be expressed as

$\frac{\partial \Phi_j}{\partial x} = 0$ at x = $M_2$ for j = 2, x = $M_3$ for j = 3, when $-d_1 < z < -(d_1+d2)$. … (4)

Zero vertical velocity on the horizontal sides of the tunnel can be expressed as

$\frac{\partial \Phi_j}{\partial z} = 0$ at z = $-d_1$ for j = 5, z = $-(d_1+d2)$ for j = 6, when $M_2 < x < M_3$. … (5)

In addition there are some continuity conditions along the vertical boundaries of the subregions. The condition for the vertical barrier can be expressed as

$\frac{\partial \varphi_3}{\partial x} = \frac{\partial \varphi_4}{\partial x} = i a_{1,0} G(\varphi_3 - \varphi_4)$, at $x = M_1$, $-h<z<0$, … (6)

where $G$ denotes the dimensionless porous-effect parameter of the porous barrier which is placed at x = $M_1$. The parameter G is in general complex which can be written in general as $G = G_r + iG_i$, where $G_r$ denotes the real part and $G_i$ denotes the imaginary part.

Which altogether give rise to a system of linear equations. By solving it, we can study the reflection coefficient.

The potential for region 1 and region 2 can be written in the following form respectively,

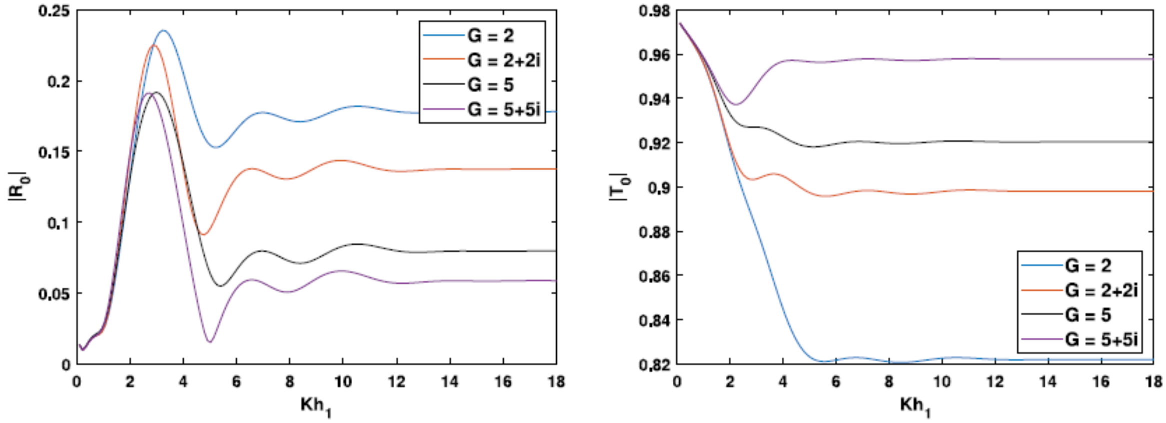
$\varphi_1(x,z) = e^{im_{1,0}x} Z_{1,0}(\alpha_{1,0}, z) + \sum_{n=0}^{\infty} R_0\, e^{-im_{1,n}x} Z_{1,n}(\alpha_{1,n}, z)$, … (7)

$\varphi_2(x,z) = \sum_{n=0}^{\infty} \left(P_{2,n} e^{im_{2,n}x} + Q_{2,n} e^{-im_{2,n}(x-M)}\right) Z_{2,n}(\alpha_{2,n}, z)$, … (8)

where $Z_{j,n}(\alpha_{j,n}, z) = \cosh \alpha_{j,n} (z + h_j)/\cosh \alpha_{j,n} h_j$, with $m_{j,n} = \sqrt{\alpha_{j,n}^2 - l^2}$ for $n = 0,1,2,\ldots,\infty$, and $R_0$ is the unknown complex reflection coefficient along with the unknowns $R_n$, $P_{2,n}$, $Q_{2,n}$. Here, $\alpha_{0,n}$ is the real root and $\alpha_{j,n}$ for $n = 1,2,\ldots,\infty$ are the purely imaginary roots of the dispersion equation $\alpha_j \tanh \alpha_j h_j = K$ for j = 1, 2.

In a similar manner, the other velocity potentials $\varphi_3, \varphi_4, \varphi_5$ and $\varphi_6$ can be obtained.

## 2. RESULTS & HIGHLIGHTS




(a) (b)

Figure 2: (a) Reflection coefficient $|R_0|$ and (b) transmission coefficient $|T_0|$ versus $Kh_1$ for various values of the porous-effect parameter G, with $h_2/h_1$ = 29/30, $h_3/h_1$ = 29/30, $h/h_3$ = 0.5, $L/h_1$ = 0.5, $L_1/h_1$ = 0.5, $L_2/h_1$ = 0.5, $L_3/h_1$ =0.5, $d_1/h_3$ = 0.25, $d_1/h_3$ = 0.15, $\theta = 30^0$.

Figure 2 presents the reflection coefficient $|R_0|$ and transmission coefficient $|T_0|$ against the non-dimensional wavenumber $Kh_1$ for different values of the dimensionless porous-effect parameter G of the barrier. As the value of $Kh_1$ increases, the reflection coefficient increases and reaches its maximum for a certain $Kh_1$. It starts decreasing for a further increment of $Kh_1$ and then gets stabilized for $Kh_1 \geq 13$. The transmission coefficient decreases as the value of G increases, and it becomes stable for $Kh_1 \geq 10$. To circumspect the matter of considering or not considering the inertial effect of G, the effect of the barrier without the inertia effect and with the inertia effect on the reflection coefficient and the transmission coefficient, and a brief comparison between them is also studied here. It is noticed that when the inertial effect is added corresponding to their respective real values of G, i.e., $G_{i,}$ the reflection coefficient becomes lower for $Kh_1 \geq 2.86$ and the transmission coefficient is significantly higher for $Kh_1 \geq 1.9$.

As shown above, the effect of various other parameters on wave reflection and transmission is also analyzed.

### REFERENCES

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2. R. Mondal, K. Takagi, “Wave scattering by a fixed submerged platform over a step bottom,” Proc. Inst. Mech. Eng. M 233 (1), 93–107, 2019.